Single-particle Foucault oscillator powered by laser

A. E. Kaplan

Dept. of Electrical and Computer Engineering,

Johns Hopkins University, Baltimore, MD 21218

(Dated: February 27, 2009)

Abstract

An ion, atom, molecule or macro-particle in a trap can exhibit large self-sustained motional oscillations due to the Doppler-affected radiation pressure by a laser, blue-detuned from a resonant absorption line of a particle. This oscillator can be nearly thresholdless, but under certain conditions it may exhibit huge hysteretic excitation. Feasible applications include a "Foucault pendulum" in a trap, a rotation sensor, single atom spectroscopy, isotope separation, etc.

PACS numbers: 42.50.Vk, 32.80.Pj, 33.80.Ps, 05.45.-a

Self-sustained oscillators (SSO) [1] are abundant in the world around us. Examples are organ and violin, sea waves, many biological processes, car engines, clocks and watches, the van-der-Pol oscillator in electronics, maser and laser, El Niño and La Niña cycles, the luminosity oscillations of many stars, and perhaps even the universe itself, to name just a few. While the energy supply (pumping) can be provided by all kinds of sources, the common facilitator of all SSO is a so called positive feedback, which overcomes damping by properly controlling the system during an oscillation cycle. A zero steady-state then becomes unstable, and the oscillations grow until they reach a stable limit cycle with a well defined amplitude and frequency of coherent oscillation. Many SSO also exhibit hysteretic excitation (bistability) and related multi-limit cycles due to nonlinearity.

It is of great interest to explore SSO in its most fundamental setting by using single particles. The "one-atom" maser [2a] and laser [2b] generate EM radiation with relatively high energy photons having quantum-mechanical statistics [2c], whereas a single particle *motional* SSO allows for nonlinear and highly excited *classical* motion and interesting applications. While trapped/cooled ions, atoms [3-6] and molecules [7] have became fascinating objects of intensive research on their quantum properties and applications [3,4], their "classical" dynamics remains somewhat less explored. In this Letter, we propose to use a trapped atom, ion, molecule, or macro-particle to excite a *motional* SSO powered by Doppler-affected radiation pressure of light blue-detuned from an atomic absorption line. This Light Activated Self-Sustained Oscillator (LASSO) is a controllable and stable system with large amplitude excitation and all the major SSO features; for certain conditions it is almost thresholdless. A LASSO could be used as a rotation sensor based on the "Foucault pendulum" in a trap, single atom spectroscopy, isotope separation, etc. The idea of using near-resonant *red−detuned* light to impose strong damping on the motion of atoms and ions via Doppler-affected radiation pressure [6] provided a powerful tool for laser cooling [3-5]. It is obvious then [8] that a *blue-detuned* laser would facilitate a Doppler instability and the positive feedback needed for SSO. For a particle in a trap, the required laser intensities are extremely low and allow for *cw* operation of a LASSO, providing for interesting applications. Blue-detuned radiation was recently proposed for use in atom waveguides and concave traps [9].

Classical motion of a particle in a 1-D harmonic potential in the z-axis with a frequency Ω which is illuminated by two EM-waves counterpropagating in the same z-axis, is governed by the equation:

$$\ddot{z} + \Omega^2 z = [F_L(z, \dot{z}, t) + F_T(z, \dot{z})]/M; \qquad (1)$$

where the "dot" designates d/dt, z is the atom location, F_T is a damping force due to losses in a trap, F_L is a light-pressure force, and M is a particle mass. A common damping factor is Stokes' drag force due to residual gas, $F_T^{(g)}/M \approx -\dot{z}|\dot{z}|/L_g$, where $L_g = (M/M_g + 1) \cdot (N_g\sigma)^{-1}/2$ is the mean free path, $(N_g\sigma)^{-1}$, scaled by a mass-factor due to energy/momentum transfer to a gas molecule of mass M_g; N_g is the gas number density, and $\sigma = \pi(d_p + d_g)^2/4$ is the cross-section of a collision particle – gas molecule of respective diameters d_p and d_g. In a static trap, another factor is the energy decay of a charged particle via lossy trap circuits [4]; here $F_T^{(t)}/M = -2\Gamma\dot{z}$, where $(2\Gamma)^{-1}$ is a relevant relaxation time.

The radiation force F_L is caused by laser beams with their frequency ω in the laboratory frame being blue-detuned from an atomic frequency ω_A. Since in most of configurations of interest, the pumping beams can be assumed weakly focused and regarded as plane waves, the scattering force due to photon absorption [3,6] (to be followed by spontaneous emission) is dominant over the gradient (or dipole, or stimulated emission) force [3,5,9], which is due to spatial inhomogeneity of each wave [10]. We consider here only the spontaneous component of F_L; while simplifying the basic theory of LASSO, this assumption is not critical. Assigning the subscript "+" to a wave propagating in the positive z direction, and "-" to the opposite direction, we have $F_L = F_+ - F_-$, where $F_\pm = \hbar k(dN_\pm/dt)$, $k = \omega/c$, and $dN_\pm/dt$ is the rate of photon absorption by the atom from the respective waves. In the case of large detunings (see below), for most part of a motional cycle, the absorption/radiation is a virtual transition to be regarded an instantaneous elastic scattering, leaving the excited level depopulated.

The atom energy losses in vacuum (and hence the required laser pumping) are low, and all the major effects emerge at intensities many orders of magnitude lower than the saturation intensity; and typically (and preferably) the trap frequency Ω is much lower than the atomic absorption linewidth, γ (the "weak binding" limit). Thus, we can use a no-saturation and "polarization-follows-the-driving" approximation, especially for large detunings $\Delta\omega = \omega - \omega_A$, when $\Omega^2 \ll \gamma^2 + \Delta\omega^2$, which is the case of most interest; its results coincide with those of a classical Lorentz absorption model. The instantaneous (on the motional scale) radiation forces are then:

$$F_\pm = (\hbar k) \cdot (\gamma\Omega_R^2)[\gamma^2 + (\Delta\omega \mp k\dot{z})^2]^{-1} \qquad (2)$$

where $\mp k\dot{z}(t)$ are instantaneous Doppler shifts of atomic frequency with regard to the "±" waves respectively (note that for large oscillations, the peak shifts, occurring near the center of oscillations, $z = 0$, are large, $k|\dot{z}_{pk}| \gg \gamma$), $\Omega_R = e\vec{d}\cdot\vec{\mathcal{E}}/\hbar$ and $\vec{\mathcal{E}}$ are the Rabi frequency and the amplitude of each wave, $e\vec{d}$ is an atomic dipole moment. The force F_L can be written as $\dot{z}\cdot\Delta\omega\cdot Q$, where $Q > 0$, e. g. $Q \approx 4(\hbar k)^2\cdot\gamma\Omega_R^2/(\gamma^2+\Delta\omega^2)^2$ for small oscillations. If $\Delta\omega > 0$, we have $F_L/\dot{z} > 0$, which makes F_L an anti-damping force. It is easily understood from the photon absorption viewpoint: as opposite to the atom cooling by red-shifted photons, when the blue-shifted photon is absorbed by an atom at its resonant frequency, the excess energy of the absorbed photon would go into kinetic energy of the atom and heat it up. If F_L/M overcomes the damping $2\dot{z}\Gamma$ in (1), the oscillations build up resulting in SSO, which is essentially a classical "squeezed" process with well determined, low-fluctuation amplitude and uncertain phase. The nonlinearity of F_L *vs* $\dot{z}$ at some point arrests this growth, and a limit cycle (a steady-state mode of SSO) is established. Thus, albeit the process can be viewed as something opposite to cooling, is not simply heating the atom; its energy is transformed into a highly ordered SSO motion, which differs from a thermal heating the same way as the sound of violin differs from a street noise, or laser radiation – from that of a filtered black-body radiation.

Using the envelope approximation [1] (applicable here since $\Gamma + a\Omega/\tilde{L} \ll \Omega$), and writing $z \approx a\sin(\Omega t + \phi)$, where $a(t)$ and $\phi(t)$ are slowly varying amplitude and phase of the oscillations respectively, we arrive at the equation for the dynamics of the peak velocity, $v(t) = \dot{z}_{pk} = a(t)\Omega$, alone:

$$\dot{v} = v\cdot[G(v^2) - \Gamma - |v|/\tilde{L}], \tag{3}$$

where $G = (2\pi M v^2)^{-1}\int_{-\pi}^{\pi} F_L(\dot{z})\dot{z}d(\Omega t)$ is the gain due to the radiation force averaged over the oscillation cycle, and $\tilde{L} = L_g(3\pi/4)$, with $2\dot{z}G$, $-2\dot{z}\Gamma$, and $-2\dot{z}|v|/\tilde{L}$ being the Fourier Ω-components of $F_L(t)/M$, $F_T^{(t)}(t)/M$, and $F_T^{(g)}(t)/M$, respectively. Using dimensionless parameters δ, ρ, and u, defined as

$$\delta = \Delta\omega/\gamma; \quad \rho = \Omega/\gamma = |\mathcal{E}ed|/\hbar\gamma; \quad u = vk/\gamma, \tag{4}$$

we evaluate the nonlinear gain G as:

$$G(\rho,\delta,u^2) = 2^{3/2}(\hbar\omega^2/Mc^2)\delta\rho^2/D(\delta,u^2) \tag{5}$$

where $D = C\sqrt{A^2 - Bu^2 + AC}$ is a nonlinear dispersion factor, with $A = \rho_{sat}^2 = 1 + \delta^2$ – a normalized saturation intensity, $B = \delta^2 - 1$, and $C = \sqrt{A^2 - 2Bu^2 + u^4}$. The *cw* mode follows from (3) with $\dot{v} = \dot{u} = 0$. One of *cw* solutions is $u = 0$. The *cw* mode with $u \neq 0$ is due to the losses exactly compensated for by the light induced gain, $G_{u\neq 0} = \Gamma + |u|\gamma/kL$, and the motional amplitude u is determined implicitly by the equation:

$$\rho^2 = 2^{-3/2}(\rho_T^2 + \rho_g^2|u|)D(\delta, u^2)/\delta \tag{6}$$

with "trapping" and "collisional" loss parameters:

$$\rho_T^2 = (Mc^2/\hbar\omega)(\Gamma/\omega), \quad \rho_g^2 = \rho_T^2/r; \tag{7}$$

and $r = (\Gamma/\gamma)k\tilde{L}$. The parameters ρ_T^2 and ρ_g^2 are tremendously lower than the saturation intensity, ρ_{sat}^2. Using as an example the *static* trapping a Na-like $^{24}Mg^+$ ion [4] with $\lambda \sim 280nm$, $d \sim 1.5 \times a_0$, where a_0 is the Bohr radius, $\gamma \sim 1.2 \times 10^8 s^{-1}$, gas of H_2 with $d_p + d_g \sim 0.2nm$, $\Gamma \sim 10^{-4}s^{-1}$ and a pressure of $7.7 \times 10^{-9}torr$, we have $\rho_T^2 \sim 4 \cdot 10^{-14}$, and $\rho_g^2 \sim 1.2 \times 10^{-15}$. The threshold pumping, ρ_{thr}^2, required to excite the LASSO in a "soft" way, i. e. from zero, $G_{u=0} = \Gamma$, is due only to ρ_T:

$$\rho_{thr}^2 = \rho_T^2 \cdot (1 + \delta^2)^2/2\delta. \tag{8}$$

It is the lowest, $\rho_{min}^2 = (2/\sqrt{3})^3\rho_T^2$, at $\delta = 3^{-1/2}$, and corresponds to the field intensity $\rho_{min}^2(\hbar\gamma/ed)^2(240\pi)^{-1} \sim 10^{-14}\ W/cm^2$, so that the LASSO here is virtually *thresholdless* (but may not be so for large detunings). The rest of the LASSO characteristics depend on the ratio $r \equiv \rho_T^2/\rho_g^2$. In our example $r \sim 34 \gg 1$, as is typical for ion trapping. For $r \gg 1$, one can neglect the collisional losses; based on (6), Fig. 1 depicts the motional amplitude u in such a case *vs* the normalized laser intensity, $I = \rho^2/\rho_{min}^2$. While a single-atom experiment is of fundamental significance, for a proof-of-principle experiment as well as application-wise, a charged macro-particle is perhaps a better candidate. A cluster of e. g. alkali or rare-gas atoms, or even a charged oil droplet, similar to the ones used by Millikan in his experiments on the measurement of an electron charge, may be easy setup that should be also furnished with a simple static trap and low-power laser. .PP In general, parameter r may vary widely (e. g. for neutral particle trapping, $r \ll 1$ under proper conditions, see below), which affects the dependence of the LASSO amplitude on the pumping intensity. However, the common and most remarkable feature emerging here for any r, is large hysteresises when the detuning and pumping are sufficiently large. The hysteresis-free domain is larger for the case of dominant collisionless losses, $r \ll 1$, since those losses are negligibly low at small

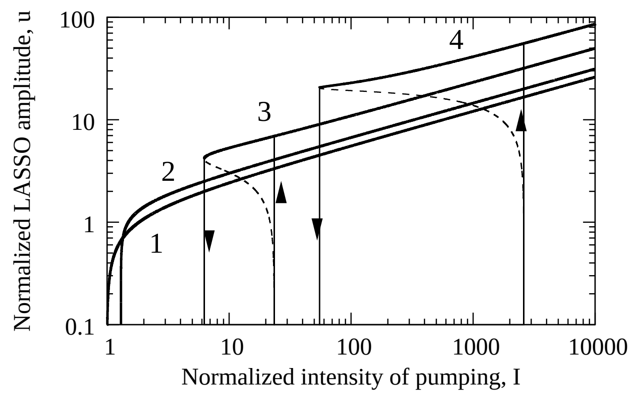

FIG. 1: The normalized LASSO amplitude (and peak velocity) u *vs* the normalized pumping, $I = \rho^2/\rho_T^2$, for $r \gg 1$. Curves: (a), 1: $\delta = 1/\sqrt{3}$, 2: $\delta = 1$, 3: $\delta = 4$, 4: $\delta = 20$. Solid lines - stable, broken - unstable modes. Arrows show the directions of hysteresis jumps.

amplitudes; in this case the hysteresis-free LASSO excitation exists for $0 < \delta < 2.75$, for any ρ, and for $\rho^2 > 16\rho_g^2$, for $\delta > 2.75$. If $r \ll 1$, a similar domain is $0 < \delta < 1$ for any ρ. At the onset of hysteresis $u \approx 2.15$ if $r \ll 1$, whereas $u \ll 1$ if $r \gg 1$ (as in the above example). Within a LASSO hysteresis loop, there are two to three non-zero steady-states, depending on r. Examination of particle dynamics using (3) shows that the limit cycle with the largest u is always stable. The same is true for the lowest u in the case of three non-zero u's (emerging at $r \ll 1$), whereas the intermediate limit cycle is unstable. With two non-zero u's (emerging at $r \gg 1$), the limit cycle with lower u is unstable. The zero-point steady-state, $u = 0$, is always stable at $\rho^2 < \rho_{thr}^2$, and unstable otherwise. The solution for the lowest branch, $u_{low}^2 \ll 1 + \delta^2$, away from the immediate vicinity of a hysteresis jump, is

$$u_{low} \approx r(\rho^2/\rho_{thr}^2 - 1). \tag{9}$$

On the upper branch, $u_{high}^2 \gg 1 + \delta^2$, in the limit $\rho_T^2 \ll u\rho_g^2$, we have $u_{high} \approx (2\delta\rho^2/rho_g^2)^{1/4}$, while in the limit $\rho_T^2 \gg u\rho_g^2$, $u_{high} \approx (2\delta\rho^2/\ \rho_T^2)^{1/3} = (2\delta I)^{1/3}\sqrt{3}/2$. The former limit is best seen on the *log-log* plots in Fig. 1. The higher the magnitude of δ and ρ^2, the larger is the loop. Evaluating the "contrast of hysteresis" as the ratio w_{hys} of the "up-jump" intensity, $\rho_u p^2$, to the "down-jump" intensity, ρ_{dwn}^2, and making use of (6) for $\delta^2 \gg 1$, we estimate that at the down-jump $u \sim \delta$ regardless of r. Thus, if $r \gg 1$, $\rho_{dwn}^2 \sim \rho_T^2 \delta^{3/2}$, while if $r \ll 1$, $\rho_{dwn}^2 \sim \rho_g^2 \delta^{5/2}$. The intensity at which the up-jump occurs is $\rho_{up}^2 \approx \rho_{thr}^2$ if $r \gg 1$. In the limit $r \ll 1$, one obtains $u \sim \delta/2$ at the onset of the up-jump. Hence $2\rho_{up}^2 \sim 3^{3/2}\rho_g^2(\delta/2)^4$.

Thus, in both limits,

$$w_{hys} \approx const \cdot \delta^{3/2}, \quad const = O(1). \tag{10}$$

Here $const \sim 0.5$ if $r \gg 1$, and ~ 0.16, if $r \ll 1$. Typically, if $\delta \gg 1$, the contrast w_{hys} is large.

The experimental conditions for LASSO observation can readily be arranged. In fact, the heating observed in [11] may signify a *transient* SSO excited during that part of the cycle of the *driven* rf side-band motional oscillations in the Paul trap, when due to *driven* Doppler shift, an ion sees the laser (on average red-detuned) as blue-detuned. However, the related micromotion is far from a well defined steady-state LASSO regime. The strong binding arrangement in [11] should produce a much more complicated picture of motion than LASSO. The hysteresis in the motion of a trapped single ion has been observed experimentally in [12a] and numerically in [12b]. The major nonlinearity in [12] comes however from $two-photon$ excitation of a *three-level* ion (e. g. Ba^+) pumped by *two* lasers with their frequencies near-resonant to different atomic transitions. The LASSO, on the other hand, involves the simplest, two-level atom model with a single-photon resonance, and is based only on the Doppler effect and not the *atomic* nonlinearity. The LASSO produces huge oscillations that take the system far beyond the Lamb-Dicke limit, $ka \ll 1$, common in trapping/cooling physics, as well as huge hysteresises with a contrast of few orders of magnitude.

To explain the LASSO hysteresis, we note that the larger the detuning, the lower is the dispersion, hence the smaller is the Doppler *positive feedback* for *small* oscillations. However, sufficiently *large* oscillations at the *same* pumping can be self-sustained, provided their peak Doppler shift u brings the atom closer to an almost *exact* resonance where the dispersion is the strongest, i. e. $|u| \sim \delta$ or $k|v| \sim \Delta\omega$, see (5). Hence, once large oscillations are excited, they can be supported even by a *lower* pumping intensity. A related feature is that if $u, \delta \gg 1$, a peak positive feedback occurs only during a short time, when the instantaneous Doppler effect (2) in *one of the waves* almost compensates the detuning, $k\dot{z}(t) \sim \pm\Delta\omega$. This is reminiscent of tennis: the game can be sustained only if the ball speed is high enough. (Note here that the LASSO hysteresis differs from that in cyclotron excitation of a single electron [13], since in the latter case we are dealing with *driven* and not SSO motion.)

Both the pumping detuning δ and intensity ρ^2 are easily controlled. Even at the extremes, the LASSO operates far below the saturation of the atomic absorption. The weak binding, $\Omega/\gamma \ll 1$ is most favorable for LASSO operation. (A strong binding will be considered by us elsewhere.) Typically, the SSO amplitude is much larger than the laser wavelength; thus the standing wave pattern [10] in F_L was justifiably neglected. It is also worth noting that while standing wave wave was used here for simplicity sake, actually the same LASSO effect can attained by using only one traveling wave [10]; the main difference is that in *cw* mode in this case the center of particle motion would not coincide with the lowest point of trap potential.

For large LASSO oscillations, the harmonicity of the trap potential in (1) would not hold, thus affecting the LASSO frequency. If the anharmonicity is symmetric, the term $\Omega^2 z$ in (1) could be replaced by $\Omega^2 z(1 \pm z^2/z_T^2)$, where z_T is the half-size of the trap; signs "-" and "+" correspond to "soft" and "hard" potentials, respectively, and the LASSO frequency is $\Omega_{NL} \approx \Omega(1 \pm 3a^2/8z_T^2)$, provided $a^2 \ll z_T^2$. A motional quantum excitation number, $(u^2/2)(\gamma/\omega)^2 Mc^2/\hbar\Omega$, for typical conditions is $10^5 - 10^7$; thus, the LASSO is a strongly classical system.

An elliptic or circular LASSO orbit in a *2-D* LASSO can be attained by using two pairs of counterpropagating laser beams, with their axes normal to each other; circular motion will require the same pairs' intensities. One of the differences of such a "LASSOtron" resonance from a magnetic cyclotron resonance [13] is that the particle does not have only one direction of revolution prescribed by a *dc* magnetic field and its charge, but may instead, depending on initial conditions, revolve now in any direction in the plane containing the beams.

While static traps allow for ion trapping [3,4] only, a neutral atom can be trapped optically by gradient (dipole) forces [5a]. It is attracted to the high-intensity areas if $\Delta\omega < 0$, and the low-intensity areas if $\Delta\omega > 0$. An all-optical LASSO is attained by gradient-trapping an atom Na, with $\lambda \sim 590nm$ on the $3s \rightarrow 3p$ D-line, $d \sim 2.2 \cdot a_0$, and $\gamma \sim 2.7 \times 10^7 s^{-1}$, in the focal area of a single red-detuned laser [5b], while LASSO-pumping (using scattering forces) is attained by an a weakly focused blue-detuned laser. The trapping laser here induces also a Doppler spontaneous damping. To use a blue-detuned laser only, one can arrange two collinear counterpropagating beams having their foci set apart [5c] and a "doughnut" [14] profile with non-zero intensity on the axis. The only damping here is due to collisions, and this LASSO is completely thresholdless.

The LASSO has promising potential as a rotation sensor based on a "Foucault pendulum" (FP) effect: a LASSO-excited (and then left alone) particle conserves its oscillation direction in space. For the FP observation in the $2-D$ configuration, one can use two degenerate degrees of freedom found in most of existing trap, to make a "Foucault plane". Small imperfections, ΔU, in the *2-D* symmetry of that potential can be made negligible by proper design and machining, as well as by choosing a heavy particle and a large LASSO amplitude, similarly to a regular FP. To this end, macro-particles offer the best possibility. (One may use here dielectric sphere Mie resonances [5] due to the coupling of the laser to the high-finesse whispering gallery modes.) The critical (lowest) rotational rate picked up by the FP, is $\Omega_{cr} \sim a^{-1}\sqrt{2e\Delta U/M}$. If $M = 1g$, $a = 1cm$, and $\Delta U \sim 10^{-5}V$, one has $\Omega_{cr}/2\pi \sim 10^{-9}Hz \sim 10^{-4}$ of the earth rotation rate, which is sufficient for inertial navigation. A slowly dissipating particle energy can repeatedly be replenished by a laser aligned to the new direction of oscillations in the lab frame.

The LASSO can be used for single particle spectroscopy by studying LASSO amplitude *vs* laser frequency. Another related application is isotope separation, whereby a laser tuned in between atomic lines of two isotopes, cools down the isotope with higher atomic frequency and LASSO-excites the other isotope, pushing it out if the pumping is sufficient. This approach can be modified and enhanced by using hysteresis and the fact that the up-jump intensity of the hysteresis is very sensitive to the detuning. With a laser blue-detuned from *both* the atomic frequencies, one can attain large LASSO-excitation of one of the species while keeping the other one at rest. Gradient force isotope separation by a focused Gaussian laser beam [5d] could also be greatly enhanced by Doppler damping (instability), which is a significant factor in pulling in (pushing out) higher (lower) atomic frequency species.

In conclusion, weak *cw* laser radiation can induce a Doppler instability and large self-sustained motional oscillations of a trapped single particle via the radiation pressure of light blue-detuned from an absorption resonance. The oscillations exhibit huge hysteresis and may easily reach a trap-size orbit. The effect can potentially be used for rotation sensing, inertial navigation, single-atom spectroscopy, and isotope separation.

This work is supported by AFOSR.

[1] See e. g. N. Minorsky, *Nonlinear Oscillations*, Princeton, NJ, Van Nostrand, 1962.

[2] (a) D. Meschede, H. Walther, and G. Müller, Phys. Rev. Lett. **54**, 551 (1985); (b) K. An, J. J. Childs, R. R. Dasari, and M. S. Feld, *ibid* **73**, 3375 (1994); (c) P. Filipowicz, J. Javanainen,

and P. Meystre, Phys. Rev. **A34**, 3077 (1986).

[3] For reviews on trapping/cooling see e. g. *Laser manipulations of atoms and ions*, Eds. E. Arimondo, W. D. Phillips, and S. Strumia (N.-Holland, Amst., 1992); H. Metcalf and P. van der Straten, Phys. Rep. **244**, 203 (1994); S. Chu, Rev. Mod. Phys., **70**, 685 (1998); C. Cohen-Tannoudji, *ibid*, 707; W. D. Phillips, *ibid*, 721.

[4] For recent review on coherent quantum-state manipulation of trapped ions see D. J. Wineland, C. Monroe, W. M. Itano, D. Leibfried, B. E. King, and D. M. Meekhof, J. of Research of NIST, **103**, 259 (1998).

[5] (a) For reviews on particle trapping by gradient forces, see A. Ashkin, Science, **210**, 1081 (1980) and Proc. Natl. Acad. Sci. USA, **94**, 4853 (1997); (b) S. Chu, J. B. Bjorkholm, A. Ashkin, and A. Cable, Phys. Rev. Lett. **57**, 314 (1986); (c) A. Ashkin and J. M. Dziedzic, *ibid* **54**, 1245 (1985); (d) J. B. Bjorkholm, R. R. Freeman, A. Ashkin, and D. B. Pearson, *ibid*, **41**, 1361 (1978).

[6] T. Hänsch and A. Schawlow, Opt. Comm. **13**, 68 (1975); V. S. Letokhov, V. G. Minogin, and B. D. Pavlik, *ibid* **19**, 72 (1976); W. Neuhauser, M. Hohenstatt, P. E. Toschek, and H. G. Dehmelt, Phys. Rev. Lett. **41**, 233 (1978); D. J. Wineland and W. M. Itano, Phys. Rev. **A20**, 152 (1979); S. Chu, L. Hollberg, J. B. Bjorkholm, A. Cable, and A. Ashkin, Phys. Rev. Lett. **55**, 48 (1985).

[7] T. Takekoshi, B. M. Patterson, and R. J. Knize, Phys. Rev. Lett. **81**, 5105 (1998).

[8] Laser-induced SSO due to Doppler effect was proposed in A. E. Kaplan, "On Doppler instability of resonantly absorbing (or amplifying) systems in a strong field," Abstr. the 6th Natl. Conf. on Nonlinear Optics (Minsk), publ. by Moscow St. Univ., Moscow, 1972, p. 26.

[9] J. P. Dowling and J. Gea-Banacloche, Adv. Atomic, Mol., and Opt. Phys. **37**, 1 (1996).

[10] We also neglect the spatial modulation of F_L with the period $\lambda/2$ mimicking a standing wave pattern; since $ka \gg 1$ as opposite to the Lamb-Dicke limit, this modulation will average out. Besides, it is present only for the standing wave and vanishes for a traveling wave pumping, J. Javanainen, M. Kaivola, U. Nielsen, O. Poulsen, and E. Riis, J. Opt. Soc. Am. B **2**, 1768 (1985).

[11] R. Blömel, C. Kappler, W. Quint, and H. Walther, Phys. Rev. **A40**, 808 (1989); E. Peik, J. Abbel, Th. Becker, von Zanthier, and H. Walther, *ibid*, **A60**, 439 (1999).

[12] (a) Th. Sauter, H. Gilhaus, I. Siemers, R. Blatt, W. Neuhauser, and P. E. Toschek, Z. Phys.

D **10**, 153 (1988); (b) J. Javanainen, in *Fundamentals of Laser Interaction. Lecture Notes in Physics 229*, Ed. F. Ehlotzky, NY, Springer (1985).

[13] A. E. Kaplan, Phys. Rev. Lett., **48**, 138 (1982) and **56**, 456 (1986); A. E. Kaplan and Y. J. Ding, IEEE JQE, **24**, 1470 (1988).

[14] K. T. Gahagan and G. A. Swartzlander, Jr., Opt. Lett. **21**, 827 (1996) and JOSA B **16**, 533 (1999).