Single-particle motional oscillator powered by laser

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Abstract: An ion, atom, molecule or macro-particle in a trap can exhibit large motional oscillations due to the Doppler-affected radiation pressure by a laser, blue-detuned from an absorption line of a particle. This oscillator can be nearly thresholdless, but under certain conditions it may exhibit huge hysteretic excitation. Feasible applications include a ”Foucault pendulum” in a trap, a rotation sensor, single atom spectroscopy, isotope separation, etc.

OCIS codes: (140.7010) Laser trapping; (350.4855) Optical tweezers or optical manipulation

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1. Introduction

Self-sustained oscillators (SSO) [1] are abundant in the world: organ and violin, sea waves, many biological processes, car engines, clocks and watches, van-der-Pol oscillator, maser and laser, El Niño and La Niña cycles, the luminosity oscillations of many stars, and perhaps even the universe itself, to name just a few. While the energy supply (pumping) can be provided by all kinds of sources, the common facilitator of all SSO is a so called positive feedback, which overcomes damping by properly controlling the system during an oscillation cycle. A zero steady-state then becomes unstable, and the oscillations grow until they reach a stable limit cycle with a well defined amplitude and frequency of coherent oscillation. Many SSO also exhibit hysteretic excitation (bistability) and related multi-limit cycles due to nonlinearity.

It is of great interest to explore SSO in its most fundamental setting by using single particles. Amazingly, the effect based on the same basic principle of blue-shifted light pressure that provides both the pumping and positive feedback, can be observed using all kind of particles, from single atom to a macro-object, e. g. submillimeter dielectric sphere, as long as the right conditions, in particular good optical resonance, are satisfied.

The ”one-atom” maser [2] and laser [3] generate optical photons with quantum-mechanical statistics [4], whereas a single particle *motional* SSO allows for highly excited *classical* motion and interesting applications. While trapped/cooled ions, atoms [5-20] and molecules [21] have became fascinating objects of research on their quantum properties and applications [5-10], their ”classical” dynamics remains somewhat less explored. In this Letter, we propose to use a trapped atom, ion, molecule, or macro-particle to excite a *motional* SSO powered by Doppler-affected radiation pressure of light blue-detuned from an atomic absorption line. This Light Activated Self-Sustained Oscillator (LASSO) is a controllable and stable system with large amplitude excitation and all the major SSO features; for certain conditions it is almost

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thresholdless. A LASSO could be used as a rotation sensor based on the "Foucault pendulum" in a trap, single atom spectroscopy, mass-spectroscopy, isotope separation, etc. The idea of using near-resonant *red − detuned* light to impose strong damping on the motion of atoms and ions via Doppler-affected radiation pressure [16-20] provided a powerful tool for laser cooling [5-15]. On the other hand, it became obvious long ago [22] that a *blue-detuned* laser would facilitate a Doppler instability and the positive feedback needed for SSO. For a particle in a trap, the required laser intensities are extremely low and allow for *cw* operation of a LASSO, providing for interesting applications. Blue-detuned radiation was proposed also for use in atom waveguides and concave traps [23].

2. Forces and equation of motion

Let us consider the simplest model of a classical particle motion in a *1-D* harmonic potential in the z-axis with a frequency Ω under the action of EM-waves counterpropagating in the same z-axis, whereby it is governed by the equation:

$$\ddot{z}+\Omega^2 z=[F_L(z,\dot{z},t)+F_T(z,\dot{z})]/M; \tag{1}$$

where the "dot" designates d/dt, z is the atom location, F_T is a damping force due to losses in a trap, F_L is a light-pressure force, and M is a particle mass. A common damping factor is a drag force due to residual rarefied gas (see e. g. [24]). In the approximation of the so called *free molecule flow* it can be roughly estimated by simply computing the collisions of a moving micro- or macro-particle with a low-temperature molecules of the gas: $F_T^{(g)}/M \approx -\dot{z}|\dot{z}|/L_g$, where $L_g = (M/M_g + 1)\cdot(N_g\sigma)^{-1}/2$ is the mean free path, $(N_g\sigma)^{-1}$, scaled by a mass-factor due to energy/momentum transfer to a gas molecule of mass M_g; N_g is the gas number density, and $\sigma = \pi(d_p + d_g)^2/4$ is the cross-section of a collision between a particle and a gas molecule of respective diameters d_p and d_g, assuming that they both are ideal spheres. In a static (ion) trap, on the other hand, an another factor is the energy decay of a charged particle via lossy trap circuits [10]; in this case $F_T^{(t)}/M = -2\Gamma\dot{z}$, where $(2\Gamma)^{-1}$ is a relevant relaxation time.

The radiation force F_L is caused by laser beams with their frequency ω in the laboratory frame being blue-detuned from an atomic frequency ω_A. Since in most of configurations of interest, the pumping beams can be assumed weakly focused and regarded as plane waves, the scattering force due to photon absorption [5-9,16-20] (to be followed by spontaneous emission) is dominant over the gradient (or dipole, or stimulated emission) force [5-9,11-15,23], which is due to spatial inhomogeneity of each wave [25]. We consider here only the spontaneous component of F_L; while simplifying the basic theory of LASSO, this assumption is not critical. Assigning the subscript "+" to a wave propagating in the positive z direction, and "-" to the opposite direction, one has $F_L = F_+ - F_-$, where $F_\pm = \hbar k(dN_\pm/dt)$, $k = \omega/c$, and $dN_\pm/dt$ is the rate of photon absorption by the atom from the respective waves. In the case of large detuning (see below), for most part of a motional cycle, the absorption/radiation is a virtual transition to be regarded an instantaneous elastic scattering, leaving the excited level depopulated. We also assume through the paper that in the case of single atoms and ions, the trap temperature is kept significantly low, which is a common situation in most of the trap experiments with micro-particles. In application to a two-level model in the case of micro-particles, we also make a realistic assumption that the trapping potential is the same for both the ground state and the excited state, i. e. that the resonance frequency is the same at any point of the trap.

The atom energy losses in vacuum (and hence the required laser pumping to overcome it) are low, and all the major effects emerge at intensities many orders of magnitude lower than the saturation intensity; and typically (and preferably) the trap frequency Ω is much lower than the atomic absorption linewidth, γ (the "weak binding" limit). Thus, one can use a no-saturation and "polarization-follows-the-driving" approximation, especially for large detunings $\Delta\omega = \omega - \omega_A$,

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when $\Omega^2 \ll \gamma^2 + \Delta\omega^2$, which is the case of most interest; its results coincide with those of a classical Lorentz absorption model. The instantaneous (on the motional scale) radiation forces are then:

$$F_{\pm} = (\hbar k) \cdot (\gamma \Omega_R^2) \left[\gamma^2 + (\Delta\omega \mp k\dot{z})^2\right]^{-1} \tag{2}$$

where $\mp k\dot{z}(t)$ are instantaneous Doppler shifts of atomic frequency with regard to the "$\pm$" waves respectively (note that for large oscillations, the peak shifts, occurring near the center of oscillations, $z = 0$, are large, $k|\dot{z}_{pk}| \gg \gamma$), $\Omega_R = e\mathbf{d} \cdot \mathbf{E}/\hbar$ and $\mathbf{E}$ are the Rabi frequency and the amplitude of each wave, and $e\mathbf{d}$ is an atomic dipole moment. For small oscillations, the force F_L can be written as

$$F_L \equiv F_+ - F_- \approx \dot{z} \cdot \Delta\omega \cdot Q, \quad \textit{where} \quad Q = \hbar k^2 \cdot \gamma \Omega_R^2/(\gamma^2 + \Delta\omega^2)^2 > 0 \tag{3}$$

If $\Delta\omega > 0$, one has $F_L/\dot{z} > 0$, which makes F_L an anti-damping force. It is easily understood from the photon absorption viewpoint: as opposite to the atom cooling by red-shifted photons, when the blue-shifted photon is absorbed by an atom at its resonant frequency, the excess energy of the absorbed photon would go into kinetic energy of the atom and heat it up. If F_L/M overcomes the damping $2\dot{z}\Gamma$ in Eq. (1), the oscillations build up resulting in SSO, which is essentially a classical "squeezed" process with well determined, low-fluctuation amplitude and uncertain phase. The nonlinearity of F_L *vs* $\dot{z}$ at some point arrests this growth, and a limit cycle (a steady-state mode of SSO) is established.

The motion of a particle attaining up to $\sim 1eV$ energy typical for a static trap for ions, corresponds to $\sim 10^4 K$ temperature. However, although the process in consideration can be viewed as something opposite to cooling, it should not be confused with simply heating the particle: if the pumping is well above threshold, its energy is transformed into a highly ordered, coherent SSO motion, which differs from a thermal heating the same way as the sound of violin differs from a street noise, or laser radiation – from that of a black-body radiation.

Using envelope approximation [1] (applicable since $\Gamma + a\Omega/\tilde{L} \ll \Omega$), i. e. $z \approx a\sin(\Omega t + \phi)$, where $a(t)$ and $\phi(t)$ are slowly varying amplitude and phase of the oscillations respectively, one arrives at the equation for the dynamics of the peak velocity, $v(t) = \dot{z}_{pk} = a(t)\Omega$, alone:

$$\dot{v} = v \cdot [G(v^2) - \Gamma - |v|/\tilde{L}], \qquad G = (2\pi M v^2)^{-1} \int_{-\pi}^{\pi} F_L(\dot{z})\dot{z}\, d(\Omega t) \tag{4}$$

where G is the gain due to the radiation force averaged over the oscillation cycle, and $\tilde{L} = L_g(3\pi/4)$; all the term in *rhs* of Eq. (4) are the Fourier Ω-components of $F_L(t)/M$, $F_T^{(t)}(t)/M$, and $F_T^{(g)}(t)/M$, respectively. Using dimensionless parameters δ, ρ, and u, defined as

$$\delta = \Delta\omega/\gamma; \;\; \rho = \Omega_R/\gamma = |e\mathbf{d} \cdot \mathbf{E}|/\hbar\gamma; \;\; u = vk/\gamma, \tag{5}$$

one evaluates an integral for the nonlinear gain G in Eq. (4) resulting in:

$$G(\rho, \delta, u^2) = 2^{3/2}(\hbar\omega^2/Mc^2)\delta\rho^2/D(\delta, u^2) \tag{6}$$

where $D = C(A^2 - Bu^2 + AC)^{1/2}$ is a nonlinear dispersion factor, with $A = \rho_{sat}^2 = 1 + \delta^2$ – a normalized saturation intensity, $B = \delta^2 - 1$, and $C = (A^2 - 2Bu^2 + u^4)^{1/2}$.

3. Stationary self-sustained oscillations

The *cw* mode follows from (4) with $\dot{v} = \dot{u} = 0$. One of *cw* solutions is $u = 0$. The *cw* mode with $u \neq 0$ is due to the losses being exactly compensated for by the light induced gain, $G_{u\neq 0} = \Gamma + |u|\gamma/kL$, and the motional amplitude u is determined implicitly by the equation:

$$\rho^2 = 2^{-3/2}(\rho_T^2 + \rho_g^2|u|)D(\delta, u^2)/\delta \tag{7}$$

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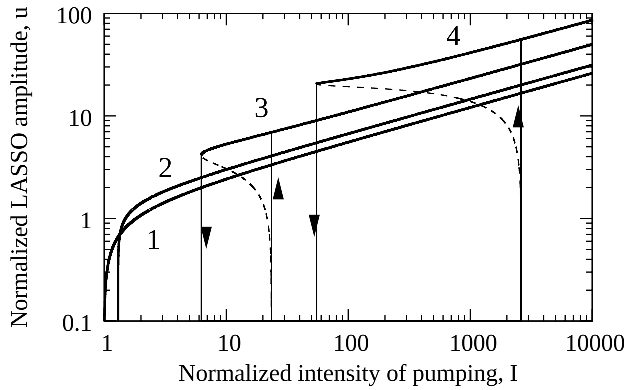

Fig. 1. The normalized LASSO amplitude (and peak velocity) u *vs* the normalized pumping, $I = \rho^2/\rho_T^2$, for $r \gg 1$. Curves: (a), 1: $\delta = 1/\sqrt{3}$, 2: $\delta = 1$, 3: $\delta = 4$, 4: $\delta = 20$. Solid lines - stable, broken - unstable modes. Arrows show the directions of hysteresis jumps.

with "trapping" and "collisional" loss parameters:

$$\rho_T^2 = (Mc^2/\hbar\omega)(\Gamma/\omega), \;\; \rho_g^2 = \rho_T^2/r; \tag{8}$$

and their ratio $r = (\Gamma/\gamma)k\tilde{L}$. The parameters ρ_T^2 and ρ_g^2 are tremendously lower than the saturation intensity, ρ_{sat}^2. Using as an example the *static* (ion) trapping a Na-like $^{24}Mg^+$ ion [10] with $\lambda \sim 280nm$, $d \sim 1.5 \times a_0$, where a_0 is the Bohr radius, $\gamma \sim 1.2 \times 10^8 s^{-1}$, gas of H_2 with $d_p + d_g \sim 0.2nm$, $\Gamma \sim 10^{-4}s^{-1}$ and a pressure of $7.7 \times 10^{-9}torr$, one has $\rho_T^2 \sim 4 \cdot 10^{-14}$, and $\rho_g^2 \sim 1.2 \times 10^{-15}$. The threshold pumping, ρ_{thr}^2, required to excite the LASSO in a "soft" way, i. e. from zero, $G_{u=0} = \Gamma$, is due only to ρ_T:

$$\rho_{thr}^2 = \rho_T^2(1+\delta^2)^2/2\delta. \tag{9}$$

It is the lowest, $\rho_{min}^2 = (2/\sqrt{3})^3\rho_T^2$, at $\delta = 3^{-1/2}$, and corresponds to the field intensity $\rho_{min}^2(\hbar\gamma/ed)^2(240\pi)^{-1} \sim 10^{-14}\ W/cm^2$, so that the LASSO is virtually *thresholdless* (but may not be so for large detunings). The rest of the LASSO characteristics depend on the ratio $r \equiv \rho_T^2/\rho_g^2$. In our example $r \sim 34 \gg 1$, as is typical for ion trapping. One can then neglect collisional losses; based on Eq. (7), Fig. 1 depicts the motional amplitude u in such a case *vs* the normalized laser intensity, $I = \rho^2/\rho_{min}^2$.

4. Multi-stable *cw* modes and hysteresis

In general, parameter r may vary widely (e. g. for neutral particle trapping, $r \ll 1$, under proper conditions, see below), which affects the dependence of the LASSO amplitude on the pumping intensity. However, the common and most remarkable feature emerging here for any r, is large hysteresises when the detuning and pumping are sufficiently large. The hysteresis-free domain is larger for the case of dominant collisional losses, $r \ll 1$, since those losses are negligibly low at small amplitudes; in this case the hysteresis-free LASSO excitation exists for $0 < \delta < 2.75$, for any ρ, and for $\rho^2 > 16\rho_g^2$, for $\delta > 2.75$. If $r \gg 1$, i. e. dominant trapping losses, a similar domain is $0 < \delta < 1$ for any ρ. At the onset of hysteresis $u \approx 2.15$ if $r \ll 1$, whereas $u \ll 1$ if $r \gg 1$ (as in the above example). Within a LASSO hysteresis loop, there are two to three non-zero steady-states, depending on r. Examination of particle dynamics using Eq. (4) shows that

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the limit cycle with the largest u is always stable. The same is true for the lowest u in the case of three non-zero u's (emerging at $r \ll 1$), whereas the intermediate limit cycle is unstable. With two non-zero u's (emerging at $r \gg 1$), the limit cycle with lower u is unstable. The zero-point steady-state, $u = 0$, is always stable at $\rho^2 < \rho_{thr}^2$, and unstable otherwise. The solution for the lowest branch, $u_{low}^2 \ll 1 + \delta^2$, away from the immediate vicinity of a hysteresis jump, is

$$u_{low} \approx r(\rho^2/\rho_{thr}^2 - 1). \tag{10}$$

On the upper branch, $u_{high}^2 \gg 1 + \delta^2$, in the limit $\rho_T^2 \ll u\rho_g^2$, one has $u_{high} \approx (2\delta\rho^2/\rho_g^2)^{1/4}$, while in the limit $\rho_T^2 \gg u\rho_g^2$, $u_{high} \approx (2\delta\rho^2/\ \rho_T^2)^{1/3} = (2\delta I)^{1/3}\sqrt{3}/2$. The former limit is best seen on the *log-log* plots in Fig. 1. The higher the magnitude of δ and ρ^2, the larger is the loop. Evaluating the "contrast of hysteresis" as the ratio w_{hys} of the "up-jump" intensity, ρ_{up}^2, to the "down-jump" intensity, ρ_{dwn}^2, and making use of Eq. (7) for $\delta^2 \gg 1$, we estimate that at the down-jump $u \sim \delta$ regardless of r. Thus, if $r \gg 1$, $\rho_{dwn}^2 \sim \rho_T^2\delta^{3/2}$, while if $r \ll 1$, $\rho_{dwn}^2 \sim \rho_g^2\delta^{5/2}$. The intensity at which the up-jump occurs is $\rho_{up}^2 \approx \rho_{thr}^2$ if $r \gg 1$. In the limit $r \ll 1$, one obtains $u \sim \delta/2$ at the onset of the up-jump. Hence $2\rho_{up}^2 \sim 3^{3/2}\rho_g^2(\delta/2)^4$. Thus, in both limits,

$$w_{hys} \approx const \cdot \delta^{3/2}, \ \ const = O(1). \tag{11}$$

Here $const \sim 0.5$ if $r \gg 1$, and ~ 0.16, if $r \ll 1$. Typically, if $\delta \gg 1$, the contrast w_{hys} is large.

To explain the LASSO hysteresis, we note that the larger the detuning, the lower is the dispersion, hence the smaller Doppler *positive feedback* for *small* oscillations. However, sufficiently *large* oscillations at the *same* pumping can be self-sustained, provided their peak Doppler shift u brings the atom closer to an *exact* resonance where the dispersion is the strongest, i. e. $|u| \sim \delta$ or $k|v| \sim \Delta\omega$, see Eq. (6). Thus, once large oscillations are excited, they can be supported even by a *lower* pumping intensity. If $u, \delta \gg 1$, a peak positive feedback occurs at the instance when the Doppler effect Eq. (2) in *one of the waves* compensates the detuning, $k\dot{z}(t) \sim \pm\Delta\omega$. This is reminiscent of the game of tennis: it can be sustained only if the ball speed is high enough. (Note here that the LASSO hysteresis differs from that in cyclotron excitation of a single electron [30-32], since in the latter case one is dealing with *driven* and not SSO motion.)

Both the pumping detuning δ and intensity ρ^2 are easily controlled. Even at the extremes, the LASSO operates far below the saturation of the atomic absorption. The weak binding, $\Omega/\gamma \ll 1$ is most favorable for LASSO operation. (A strong binding will be considered by us elsewhere.) Typically, the SSO amplitude is much larger than the laser wavelength; thus the standing wave pattern [24] in F_L was justifiably neglected. It is also worth noting that while standing wave wave was used here for simplicity sake, actually the same LASSO effect can attained by using only one traveling wave [25]; the main difference is that in *cw* mode in this case the center of particle motion would not coincide with the lowest point of trap potential.

For large LASSO oscillations, the harmonicity of the trap potential in Eq. (1) would not hold, thus affecting the LASSO frequency. If the anharmonicity is symmetric, the term $\Omega^2 z$ in Eq. (1) could be replaced by $\Omega^2 z(1 \pm z^2/z_T^2)$, where z_T is the half-size of the trap; signs "-" and "+" correspond to "soft" and "hard" potentials, respectively, and the LASSO frequency is $\Omega_{NL} \approx \Omega(1 \pm 3a^2/8z_T^2)$, provided $a^2 \ll z_T^2$. A motional quantum excitation number, $(u^2/2)(\gamma/\omega)^2 Mc^2/\hbar\Omega$, for typical conditions is $10^5 - 10^7$; thus, the system may be regarded as strongly classical at low frequencies. However, at the "optical end", under certain conditions it may become strongly quantum, since the absorption of photons by the atom, and resulting spontaneous emission of photons in all possible direction may result in the spontaneous deflection of atom from a classically-prescribed trajectory, which may be regarded as a broad-band noise induced by optical pumping. Near the threshold of oscillations, this noise will result in the broadening of the spectral line of oscillations, similar to any other self-sustained oscillator,

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such as e. g. shot noise in electronic SSO, or spontaneous radiation in laser near the threshold of its excitation. However, similarly to those system, one should expect that as the pumping significantly exceeds that threshold, $\rho^2 \gg \rho_{thr}^2$, the spontaneous noise becomes insignificant, and the oscillations will be strongly coherent and their linewidth – drastically reduced.

An elliptic or circular LASSO orbit in a *2-D* LASSO can be attained by using two pairs of counterpropagating laser beams, with their axes normal to each other; circular motion will require the same pairs' intensities. One of the differences of such a "LASSOtron" resonance from a magnetic cyclotron resonance and its hysteresises [26-28] is that the particle does not have only one direction of revolution prescribed by a *dc* magnetic field and its charge, but may instead, depending on initial conditions, revolve now in any direction in the plane of the beams.

5. Traps and settings

The experimental conditions for LASSO observation can be arranged based on the existing traps. In fact, the heating observed in [29,30] may indicate a *transient* SSO excited during that part of the cycle of the *driven* rf side-band motional oscillations in the Paul trap, when due to *driven* Doppler shift, an ion sees the laser (on average red-detuned) as blue-detuned. However, the related micromotion is far from a well defined steady-state LASSO regime. The strong binding arrangement in [29,30] should produce a much more complicated picture of motion than LASSO. The hysteresis in the motion of a trapped single ion has been observed experimentally in [31] and numerically in [32]. The major nonlinearity in [31,32] comes however from $two-photon$ excitation of a *three-level* ion (e. g. Ba^+) pumped by *two* lasers with their frequencies near-resonant to different atomic transitions. The LASSO, on the other hand, involves the simplest, two-level atom model with a single-photon resonance, and is based only on the Doppler effect and not the *atomic* nonlinearity. The LASSO produces huge oscillations that take the system far beyond the Lamb-Dicke limit, $ka \ll 1$, common in trapping/cooling physics, as well as huge hysteresises with a contrast of few orders of magnitude.

The issue of the most appropriate static (ion) trap for the LASSO experiment has to be considered separately, with recent developments in the field offering a wide range of possible traps [37]. The simplest way to observe LASSO could be to use the Penning trap, with the particle self-sustained oscillation exited along the axis of symmetry of the trap, i. e. between the trap caps and normally to the ring electrode. The main condition then is that the cyclotron frequency of a particle in a strong *dc* magnetic field be much greater than the vibrational LASSO frequency Ω, but this is typical for Penning traps [10,37]. This arrangement can be used for mass-spectroscopy and isotope separation; however it cannot support Foucault oscillator, since the LASSO motion here has strongly preferred oscillation axis. On the other hand, the Paul trap makes it feasible for a particle to choose its main direction of oscillations in a plane normal to the trap axis, and thus it could be appropriate for realization of *2-D* Foucault LASSO oscillator, although a *1-D* model implied by Eq. (1) will be a rough approximation to the full motion.

While a single-atom experiment is of fundamental significance, for a proof-of-principle experiment a charged macro-particle in the Penning of Paul trap is perhaps a better, and much more attainable and controlled candidate. (Notice, e. g. the use of an aluminum dust to demonstrate trapping in many experiments [37].) A cluster of e. g. alkali or rare-gas atoms, a dielectric or metallic charged sphere, or even a charged oil droplet, similar to the ones used by Millikan in his experiments on the measurement of an electron charge, may become an easy setup with a simple static trap for a charged particle and a low-power laser. In this case, for the pumping exceeding the threshold by a few times, the entire situation is purely classical.

While static traps allow for ion trapping [5-9,10,37] only, a neutral atom can be trapped optically by gradient (dipole) forces [11]. It is attracted to the high-intensity areas if $\Delta\omega < 0$, and the low-intensity areas if $\Delta\omega > 0$. An all-optical LASSO is attained by gradient-trapping

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an atom Na, with $\lambda \sim 590nm$ on the $3s \rightarrow 3p$ D-line, $d \sim 2.2 \cdot a_0$, and $\gamma \sim 2.7 \times 10^7 s^{-1}$, in the focal area of a single red-detuned laser [12], while LASSO-pumping (using scattering forces) is attained by an a weakly focused blue-detuned laser. The trapping laser here induces also a Doppler spontaneous damping. To use a blue-detuned laser only, one can arrange two collinear counterpropagating beams having their foci set apart [13] and a "doughnut" [33,34] profile with non-zero intensity on the axis. The only damping here is due to collisions, and LASSO here is completely thresholdless.

In a future research, it may also be worth considering the case of an atom submitted to both a blue and a red-detuned laser, so as to have at the same time damping by Doppler cooling and a LASSO effect. By using different detunings, powers, and transitions (different linewidth), it might be possible to find a regime where the velocity-dependence of the damping and the LASSO forces are different enough so as to generate stable oscillations.

6. "Foucault oscillator" and other feasible applications

The LASSO has an interesting potential as a rotation sensor based on a "Foucault pendulum" (FP) effect: a LASSO-excited (and then left alone) particle conserves its oscillation direction in space. Along with a gyroscope and an EM Sagnac effects, the Foucault pendulum [35] is one of very few effects/devices, which unequivocally demonstrate that the universe affects our "regular" mechanics and electrodynamics. However, unlike the other two effects, the Foucault pendulum needs a cathedral-size structure for its realization, which severely limits its applications for e. g. rotation detection and inertial navigation; besides, it relies on the gravitation, which imposes even stronger limits on these applications. The need in very long pendulum string comes out of the requirement for a sufficiently long relaxation time. Lowering the frequency of the pendulum makes this time longer even for the same finesse of the system; furthermore, the damping per cycle is lower for higher ratio of the string length to its diameter; hence the long pendulum. The gravitation/acceleration environment simply makes the pendulum possible (for example, the Foucault pendulum – or any pendulum – would be impossible in the space). Both these drawbacks of the Foucault pendulum can be overcame by using a single particle trapped in a symmetrical *3-D* or *2-D* trap in a vacuum. This would allow one to have a small-size motional oscillator with very low damping (which can further be completely eliminated by a low-intrusive positive feedback provided by a few laser beams, see below), whose oscillation and momentum-conservation properties will be "portable" in the sense that they be provided by the trap potential, and not by the local gravity. All this may put the LASSO used as a "Foucault trap oscillator" into the realm of inertial navigation applications similar to those of mechanical and laser (i. e. Sagnac-based) gyroscopes.

For the FP observation in the *2-D* configuration, one can use two degenerate degrees of freedom found in some of existing trap, to make a "Foucault plane". Small imperfections, ΔU, in the *2-D* symmetry of that potential can be made negligible by proper design and machining, as well as by choosing a heavy particle and a large LASSO amplitude, similarly to a regular FP. To this end, macro-particles offer the best possibility. Similarly to [11], to optically exert a light-pressure on a macro-particle, one may use very narrow Mie-Debay resonances [36] of dielectric sphere, which are due to the coupling of the laser to the high-finesse whispering gallery modes. The critical (lowest) rotational rate picked up by the FP, is $\Omega_{cr} \sim a^{-1}(2e\Delta U/M)^{1/2}$. If $M = 1g$, $a = 1cm$, and $\Delta U \sim 10^{-5}V$, one has $\Omega_{cr}/2\pi \sim 10^{-9}Hz \sim 10^{-4}$ of the earth rotation rate, which is sufficient for inertial navigation.

A slowly dissipating particle energy can repeatedly be replenished by a laser aligned to the new direction of oscillations in the lab frame. As was noted above, in principle, the random character of the exciting force (spontaneous photons emitted in all directions) may cause momentum diffusion, including in the plane perpendicular to the oscillation. However this might

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be a problem only in the case of micro-particle, and only near the threshold of the LASSO-oscillations; with increased pumping, the ratio fluctuation/amplitude will go down. And this factor can be completely ignored in the case of macro-particle, as in the above example. No regular classical Foucault pendulum even in non-rarefied air has ever been known for quantum-induced momentum diffusion.

The LASSO can be used for single particle spectroscopy by studying LASSO amplitude *vs* laser frequency. Another related application is isotope separation, whereby a laser tuned in between atomic lines of two isotopes, cools down the isotope with higher atomic frequency and LASSO-excites the other isotope, pushing it out if the pumping is sufficient. This approach can be modified and enhanced by using hysteresis and the fact that the up-jump intensity of the hysteresis is very sensitive to the detuning. With a laser blue-detuned from *both* the atomic frequencies, one can attain large LASSO-excitation of one of the species while keeping the other one at rest. Gradient force isotope separation by a focused Gaussian laser beam [14] could also be greatly enhanced by Doppler damping (instability), which is a significant factor in pulling in (pushing out) higher (lower) atomic frequency species.

7. Conclusion

In conclusion, weak *cw* laser radiation can induce a Doppler instability and large self-sustained motional oscillations of a trapped single particle via the radiation pressure of light blue-detuned from an absorption resonance. The oscillations exhibit huge hysteresis and may easily reach a trap-size orbit. The effect can potentially be used for rotation sensing, inertial navigation, single-atom spectroscopy, mass-spectroscopy, and isotope separation.

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