Bistable reflection of light by an electro-optically driven interface

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A theory of nonresonant optical bistability, based on "hybrid" interface effect, is developed. This effect occurs upon reflection of light at a single interface of a medium driven by electro-optical feedback. The nonlinear Snell's and Fresnel's formulas for the transmission regime are obtained, as well as conditions for bistable operation. The stability of steady states and characteristics of hysteresis jumps are examined as well.

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In our previous articles[1,2,6] the theory of new effects in nonlinear optics was developed, which describes, in particular, bistable (hysteresis) reflection and refraction of strong plane waves from the surface of a nonlinear medium; this effect is valid for "positive" nonlinearity[1] as well as for "negative" nonlinearity.[2] Very recently it was experimentally observed by Smith, Hermann, Tomlinson, and Maloney[3] for the case of "positive" nonlinearity.

In contrast to well-known bistable optical devices,[4] whose bistability is due to the presence of a Fabry-Perot resonator filled with nonlinear medium, the concerning phenomena is nonresonant and might use a broad spectrum of light. However, for the usual nonlinear medium (for example, Cs_2), the required intensity of incident field is close to the damage threshold. On the other hand, using the bounded beams provides a strong spatial localization of the nonlinear component of susceptibility (if that one depends on the intensity of light in the given point of space), which should lead to the secondary effects such as self-focusing and self-bending of beams as it was pointed out in Ref. 1. These effects can probably eliminate the main phenomenon of bistability in some situations. To avoid these problems the new modification of nonlinear reflection effect was proposed by us.[5,6] In this modification the electro-optical element performs as an "artificial" nonlinear medium in the system supplied with electronic feedback. The electro-optical element should be used as one of the media forming the interface (see Fig. 1). Then the light reflected from the interface and received by the detector can be transformed into an electrical signal, and the voltage resulting from this signal should be applied to the electro-optical element producing the change in its susceptibility (in general, the driving signal might be amplified by electronic means). Such a light-feedback method is analogous to the one used in "hybrid" devices,[7] where an electro-optical element is used as a driving medium in the Fabry-Perot interferometer to create an "artificial" Felber-Marburger[8] resonator. However, in our case, the driving signal changes the refraction angle of the light behind the interface rather than the phase shift of the wave in the resonator. The electro-optic element can change susceptibility of the same value practically in the entire working volume of the medium; this is the reason why the plane wave theory can give a satisfactory description of this situation.

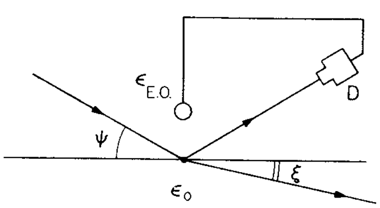

FIG. 1. Schematic diagram of a bistable device using an electro-optically driven interface; D detector of light.

Now we are developing for the first time a theory of such electro-optical interface bistability. It can be easily proven that the only situation in which hysteresis behavior of the system can exist is the one where the detector receives the reflected rather than the refracted light. In such a case, if the susceptibility of the electro-optical medium increases under an applied signal, this must be the input medium (see Fig. 1), and must have "positive" linear mismatch $\Delta\epsilon_L$, i.e., its susceptibility ϵ_{EO} in the simplest case can be written as

$$\epsilon_{EO}(I_r) = \epsilon_o + \Delta\epsilon_L + kI_r, \tag{1}$$

where ϵ_o is the susceptibility of the linear medium, $\Delta\epsilon_L > 0$ is the constant difference between susceptibilities media, which does not depend on light intensity, I_r is the intensity (or total power) of the refracted light beam, and $k > 0$ is the constant and depends upon detector (and, possible, amplifier) and electro-optic element characteristics. In the case if $k < 0$, the electro-optic element should perform as a reflected medium and the value of $\Delta\epsilon_L$ should be chosen to be negative, i.e., $\Delta\epsilon_L < 0$, the theory presented below remains valid for this case as well.

Let us assume that the incident light beam is "close enough" to a plane wave, which means in our case, that its angle of diffraction divergence is much smaller than the critical angle ψ_{cr} of total internal reflection (TIR), which can be specified by relationship $\sin^2\psi_{cr} = |\Delta\epsilon_L|/\epsilon_o$. Then, for the "transmission" wave regime (i.e., non-TIR) we can write the conventional Snell's formula for angles:

$$\left(\frac{\cos\psi}{\cos\xi}\right)^2 = \frac{\epsilon_o}{\epsilon_{EO}(I_r)}, \tag{2}$$

where ψ is the glancing angle of incidence and ξ is the angle of refraction (see Fig. 1) and Fresnel's formula for the amplitude coefficient of reflection $r = (I_r/I_{in})^{1/2}$ (where I_{in} is intensity of incident light), which differs for the cases when electric field polarization is either perpendicular (⊥) or parallel (||) to the plane of incidence:

$$r_\perp = \sin(\psi - \xi)/\sin(\psi + \xi),$$
$$r_\parallel = \mathrm{tg}(\psi - \xi)/\mathrm{tg}(\psi + \xi). \quad (3)$$

As is usual for interface nonlinear effects[1,2] in the case of a small nonlinear component ($rI_r \ll \epsilon_o$), the only conditions under which all effects being of interest can be observed are the proximity of susceptibilities and the almost-grazing incidence, i.e.,

$$\epsilon_o \gg \Delta\epsilon_L \ (\sim rI_{\rm in}),\ 1 \gg \psi,\xi. \quad (4)$$

Then, Fresnel's formulas (3) become the same for different polarizations:

$$r \equiv (I_r/I_{\rm in})^{1/2} \approx (\psi - \xi)/(\psi + \xi), \quad (5)$$

and Snell's formula (2) can be written as

$$\psi^2 - \xi^2 = (kI_r/\epsilon_o) + \psi_{\rm cr}^2, \quad \psi_{\rm cr}^2 \approx \Delta\epsilon_L/\epsilon_o. \quad (6)$$

Considered together, equations (5) and (6) yield the "nonlinear Snell's formula" for refracted angle ξ (in the case when $\psi > \psi_{\rm cr}$)

$$kI_{\rm in}/\epsilon_o = [(\psi + \xi)/(\psi - \xi)]^2(\psi^2 - \xi^2 - \psi_{\rm cr}^2), \quad (7)$$

or "nonlinear Fresnell's formula" for coefficient of reflection r:

$$kI_{\rm in}/\Delta\epsilon_L = 1/r[(\psi/\psi_{\rm cr})^2\, 4/(l + r)^2 - 1/r], \quad (8)$$

where $I_{\rm in}$ is a given intensity of incident light. Calculations for nonlinear TIR show that if $\psi < \psi_{\rm cr}$, the regime of TIR exists for all possible intensities $I_{\rm in}$, but if $\psi > \psi_{\rm cr}$, then TIR can exist only under condition

$$kI_{\rm in}/\Delta\epsilon_L > (\psi^2/\psi_{\rm cr}^2) - 1, \quad \psi > \psi_{\rm cr}. \quad (9)$$

Examination of relationship (8) shows that under condition (9) there is a range of values of input intensity $I_{\rm in}$, where bistability of states and hysteresis jumps between transmission regime and TIR arise, which is demonstrated in Fig. 2. Let us specify values of some important points at the hysteresis curves following from Eq. (8). Coefficient r at weak field ($I_{\rm in} = 0$) is specified as

$$r_o = (q - \sqrt{q^2 - 1})^2, \quad (10)$$

where

$$q \equiv \psi/\psi_{\rm cr} > 1.$$

The value of critical input intensity $I_{\rm in}$, which causes a jump from TIR state (r = 1) to transmission regime, is specified by relationship (9), where one has to take notice of equality. Maximum possible values of coefficient r_2, as well as input intensity $I_{\rm in}$ for transmission regime (the values correspond to condition $dr/dI_{\rm in} = \infty$ which provides a jump from transmission state to TIR) are specified by equations

$$2q^2r_2(1 + 3r_2) = (1 + r_2)^3, \quad q > 1 \quad (11)$$

and

$$I_{\rm in_2} = \Delta\epsilon_L/k\ [(1 - r_2)/r_2^2\ (1 + 3r_2)]; \quad (12)$$

so, if $q^2 >> 1$, then $r_2 \approx (2q^2)^{-1}$ and $I_{\rm in_2} \approx (\Delta\epsilon_L/k)\, 4q^4$

In order to examine a stability of two possible states r of transmission regime, specified by equation (8) in the range of bistability, i.e., $I_{\rm in_1} < I_{\rm in} < I_{\rm in_2}$, let us take into consideration

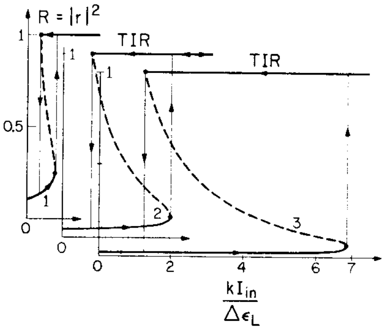

FIG. 2. Reflectivity $R = |r|^2$ as a function of input intensity $I_{\rm in}$ for different ratios $q = \psi/\psi_{\rm cr}$. Curves: $1 - q = 1.16$; $2 - q = 1.33$; $3 - q = 1.5$.

a relaxation in the system of the detector electro-optical element. In the simplest case this relaxation can be described by a linear equation of the kind

$$\tau\frac{d\Delta\epsilon_{\rm EO}}{dt} + \Delta\epsilon_{\rm EO} = kI_r, \quad (13)$$

where relaxation time τ depends upon detector and electro-optic element respnse decay characteristics (we assume that τ is the largest relaxation time in the entire system) and $\Delta\epsilon_{\rm EO}$ is a driven component of susceptibility of the electro-optical element. Substituting the value of $\Delta\epsilon_{\rm EO}$ into Eq. (6) in place of kI_r and considering Eq. (5), (6), and (15) all together, one can obtain a differential equation, describing the dynamics of parameters of transmission regime. For instance, for reflection coefficient r it yields

$$\tau\frac{dr}{dt} + r\left(\frac{1 + r}{1 - r}\right)\left(1 - \frac{[1 + r^2(kI_{\rm in}/\Delta\epsilon_L)](1 + r)^2}{4q^2r}\right) = 0 \quad (14)$$

which has a steady ($dr/dt = 0$) state solution $r_{\rm SS}$, defined by relationship (8). In order to examine the stability of these states relative to small perturbations $\Delta r = r - r_{\rm SS}$, Eq. (14) can be linearized near steady states. This new linear equation for small perturbations Δr yields solutions which can be written in the following form:

$$\Delta r \sim \exp[-\alpha t(dr_{\rm SS}/dI_{\rm in})^{-1}],$$
$$\alpha = (k/\tau\Delta\epsilon_L)\, r_{\rm SS}(1 + r_{\rm SS})^3/4q^2(1 - r_{\rm SS}) > 0, \quad (15)$$

where $r_{\rm SS}(I_{\rm in})$ is specified by (8) Now, it can be seen clearly from (15) that the stability of steady states $r_{\rm SS}$ depends only on the sign of derivative $dr_{\rm SS}/dI_{\rm in}$: the state is stable if this derivation is positive, and unstable if it is negative. Therefore the solid branches of curves $R\ (I_{\rm in})$ in Fig. 2 correspond to the stable states and dashed branches correspond to the unstable states, which is in good agreement with physical expectation.

In general, an experimental setup as well as a device for application can be built by using only a detector without any amplifiers. If a $LiNbO_3$ crystal is used as the electrooptical element geometrically analagous to that one used by Kaminov *et al.*,[9] then a value of $\Delta\epsilon_{\rm EO} \sim 3.10^{-5}$ V can be achieved, where V is applied voltage. Further, if $\Delta\epsilon_L$ is chosen to be in the order of 3.10^{-4} and an avalanche photodiode is operating as a detector, giving a sensitivity in the order of 10^2 A/W with a 100 Ω resistor used as a load, then the input light power which is necessary for bistable operation should be in the order of $I_{cr} \sim 10^{-3}$ W, which gives a considerably low level of operation. In this case, the critical angle of TIR is $\psi_{cr} \sim 1°$, and the relaxation time (if the capacity of the electrodes is ~ 10 pF) is $\tau \sim 1$ nsec. This time can be decreased by reducing load resistance, which leads to proportional increasing of critical input power.

Let us point out some advantages of the proposed device. (i) In contrast to resonator schemes, it is not sensitive to the change of wavelength and can use a broad spectrum of incident light. (ii) It can demonstrate spacial bistability, which implies bistable change of angle of a refracted beam. (iii) Switching time is limited only by the relaxation time of a network of detector-electrodes, but not by resonator response time. (iv) The length of interaction of a light beam with an interface can be made shorter than the usual length of resonators; therefore, this device can be constructed in a very small amount of space and, eventually, it can be constructed in an integrated self-contained device, analagous to Ref. 10, by using photovoltaic detectors which require no external electrical energy. (v) In contrast to resonator "hybrid" devices which are multistable under strong signal, the device described is truly bistable.

All these reasons confirm that this device is the simplest for bistable nonresonant operations and optical signal processing, such as optical memory, switching and binary logic operations, especially low level of power of input light signal is required.

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