Enhancement of the Sagnac effect due to nonlinearly induced nonreciprocity

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We propose a novel way to enhance the Sagnac effect by using a nonlinear ring interferometer. Specifically, we take advantage of the nonlinearly induced nonreciprocity of counterpropagating waves caused by the formation of an index grating in the nonlinear medium.

It is well known that the Sagnac effect[1] provides a powerful way to measure rotation rates with extreme accuracy. In particular, recent proposals to check the curvature of space and the Lense–Thirring effect are directly based on this idea.[2]

In this Letter, we propose a novel way to enhance the Sagnac effect by use of a nonlinear ring interferometer. Specifically, we take advantage of the nonlinearly induced nonreciprocity of counterpropagating waves, i.e., the different speeds of propagation of counterpropagating waves of different intensities caused by the formation of an index grating in the nonlinear medium.

The Sagnac effect is caused by the nonreciprocity (different optical path lengths) of two counterpropagating waves that is due to the rotation of a ring interferometer. In a resonator filled with a nonlinear medium, the optical path lengths are further varied as functions of the light intensity. If this nonlinear change is the same for both waves, it has no influence on the Sagnac effect. However, this need not be the case. If a nonlinear index grating is generated by standing waves, the nonlinear change of optical length is different for the left- and right-propagating waves, provided that these waves have different intensities.

Therefore we can use the initial small change of amplitude provided by the usual Sagnac effect (which is due to the dependence of amplitude on the resonance frequency detuning) to generate a large additional change through the nonlinearly induced nonreciprocity of the optical paths of the counterpropagating waves. (One of the amplitudes decreases while the other one increases, leading to nonsymmetrical propagation.) Clearly, this nonlinear enhancement is expected to increase dramatically in the vicinity of a possible nonsymmetrical instability of the counterpropagating waves.

We consider a passive ring resonator filled with a Kerr-like nonlinear medium (see Fig. 1). This system is pumped in both directions by two incident beams of same frequency ν and amplitude E_I. The nonlinear susceptibility is given by $\epsilon_{NL}(x) = \epsilon_0 + \Delta\epsilon^{NL}$, where in the simplest case $\Delta\epsilon^{NL} = \epsilon_2|E(x)|^2$. Here $E(x)$ is the total field at location x. It is well known that in the presence of two counterpropagating waves, the nonlinear component $\Delta\epsilon^{NL}$ is different for opposite directions, namely, for a Kerr-type nonlinearity,[3]

$$\Delta\epsilon_1{}^{NL} = \epsilon_2(|E_1|^2 + 2|E_2|^2),$$
$$\Delta\epsilon_2{}^{NL} = \epsilon_2(|E_2|^2 + 2|E_1|^2). \tag{1}$$

Here E_1 and E_2 label the clockwise- and anticlockwise-propagating fields, respectively, such that the total field is

$$E(x) = E_1(x)e^{ikx} + E_2(x)e^{-ikx}. \tag{2}$$

The nonreciprocal factors in Eq. (1) result from the formation of a nonlinear index grating that leads to the cross interaction of E_1 and E_2, as can be shown from the following argument. The nonlinear part of the polarization is given by $P^{NL} \propto E\Delta\epsilon^{NL}$. When Eq. (2) is used, in the case of the cubic nonlinearity one obtains

$$\begin{aligned} P^{NL} \propto E|E|^2 &= (E_1e^{ikx} + E_2e^{-ikx})[(|E_1|^2 + |E_2|^2) + (E_1E_2{}^*e^{2ikx} + \text{c.c.})] \\ &= E_1e^{ikx}(|E_1|^2 + 2|E_2|^2) \\ &\quad + E_2e^{-ikx}(|E_2|^2 + 2|E_1|^2) \\ &\quad (+\text{ rapidly varying terms}), \end{aligned}$$

so the relevant susceptibilities for the clockwise and

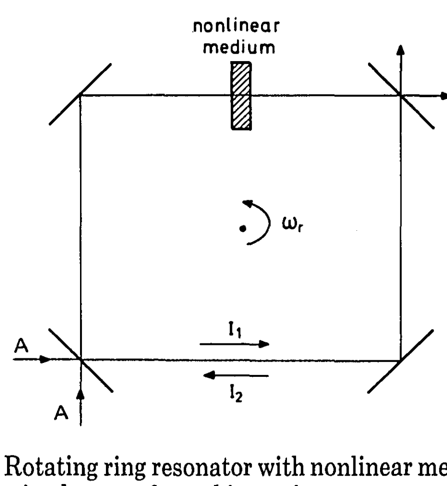

Fig. 1. Rotating ring resonator with nonlinear medium and two pumping beams of equal intensity.

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anticlockwise fields are as given in Eq. (1). We would like to stress two important facts in regard to this result. First, nonlinearly induced nonreciprocity (1) is solely due to formation of a nonlinear index grating that is caused by the periodic term $(E_1 E_2^* e^{2ikx} + \text{c.c.}) \propto \cos 2kx$ in $|E|^2$. If this periodic grating is washed out by some process (e.g. diffusion), then nonreciprocity in Eq. (1) should disappear. Second, cross interaction (1) caused by the Kerr-nonlinear index grating, does not lead to the mutual backreflection of both of the counterpropagating waves at the grating, as has been explicitly shown by Zeldovich.[3] Hence the effect discussed in the present Letter is caused solely by nonreciprocal change of optical length, not by the induced reflection.

Therefore, taking the frequency ν of the driving field to be close to one of the eigenfrequencies ν_0 of the resonator, the steady-state amplitude E_j of the field j ($j = 1, 2$) is

$$E_j = \frac{E_I(\gamma_c/\gamma T)}{1 + i\gamma^{-1}[\nu - \nu_0 + (-1)^j \omega_s + \nu_0 \Delta\epsilon_j^{\mathrm{NL}} L_s/L]}. \tag{3}$$

Here, E_I is the incident field, T is the mirror transmittivity, L is the total optical length of the resonator, and L_s is the optical length of the nonlinear sample. $\gamma_c = cT/L$ is the empty cavity bandwidth and γ_s gives the linear losses of the medium inside the cavity, so $\gamma = \gamma_c + \gamma_s$ gives the total bandwidth of the system, $k_0 = \omega_0/c$, and

$$\omega_s = 4S\omega_r k_0/L \tag{4}$$

is the Sagnac frequency shift, where S is the area of the ring resonator and ω_r is its rotation frequency. Note that Eq. (3) is an approximate form valid for the good-finesse case, that is, $\gamma \ll c/L$. We introduce the dimensionless frequencies $\Delta = (\nu - \nu_0)/\gamma$ and $\Omega = \omega_s/\gamma$ and the dimensionless intensities

$$I_j = \epsilon_2 |E_j|^2 L_s \nu_0/L\gamma, \qquad A = \gamma_c \epsilon_2 |E_I|^2 L_s \nu_0 / L\gamma T.$$

(Note that I_j, A, and Δ change sign for $\epsilon_2 < 0$.)

The equation for the intensity I_j is readily obtained by multiplying Eq. (3) by its complex conjugate:

$$A = I_j \{1 + [\Delta + (-1)^j \Omega + I_j + 2I_{3-j}]^2\}. \tag{5}$$

We assume that in the absence of rotation the intensities I_1 and I_2 are equal (reciprocal regime). That is, $I_1 = I_2 = I_0$, where I_0 satisfies the equation

$$A = I_0 [1 + (\Delta + 3I_0)^2]. \tag{6}$$

We also restrict ourselves to the most interesting case of small rotations ($\Omega \ll 1$), so we can assume that the perturbations $\zeta_j = I_j - I_0$ of both waves are small ($\zeta_j \ll I_0$). Under these conditions, we immediately find the solution for ζ_j:

$$\zeta_1 = -\zeta_2 = (\eta - 1)\Omega,$$

where

$$\eta = \frac{1 + (\Delta + 3I_0)^2}{1 + (\Delta + 3I_0)(\Delta + I_0)}. \tag{7}$$

Therefore the total phase difference between the two counterpropagating waves is given by $\eta\Omega$ instead of by Ω. Hence the result of the nonlinear medium is to scale the Sagnac effect by a factor of η. Equation (7) is an implicit form for η, and I_0 still must be determined from Eq. (6). One expects η to be much larger than unity for values of the driving field and detuning such that the system is near the onset of instability.

Calculations based on Eqs. (6) and (7) show that the boundary of the unstable regime ($\eta \to \infty$) is given by

$$A = (2/3)[-\Delta(3\Delta^2 - 5) \pm (3\Delta^2 - 1)(\Delta^2 - 3)^{1/2}]. \tag{8}$$

As is shown in Ref. 4, this instability leads to a new steady-state regime such that $I_1 \neq I_2$ (nonreciprocal regime) in the absence of the ring interferometer.

The threshold of this instability occurs for $A_{\mathrm{th}} = 8/\sqrt{3}$ and $\Delta_{\mathrm{th}} = -\sqrt{3}$, which gives $I_0 = 2/\sqrt{3}$. We find that the enhancement factor η increases dramatically as one approaches the instability boundary from the symmetrical regime of operation, i.e., as one decreases Δ. Note that for $\Delta > \Delta_{\mathrm{th}}$, the regime of operation for $\Omega = 0$ is symmetrical and stable for any value of the driving field.

In Fig. 2, we show the enhancement factor η as a function of the driving field A and for various detunings $\Delta \geqq \Delta_{\mathrm{th}}$. These curves were obtained by solving Eqs. (6) and (7) consistently. As expected, η becomes infinite for $\Delta = \Delta_{\mathrm{th}}$ and $A = A_{\mathrm{th}}$. It follows from Eq. (7) that, for a fixed detuning, the highest enhancement is given by

$$\eta_{\max} = 1/[2 - (\Delta^2 + 1)^{1/2}]. \tag{9}$$

In the vicinity of the threshold ($\Delta - \Delta_{\mathrm{th}} \ll |\Delta_{\mathrm{th}}|$), Eq. (9) reduces to $\eta_{\max} \cong 1/\sqrt{3}(\Delta - \Delta_{\mathrm{th}})$. The corresponding amplitude of the driving field is

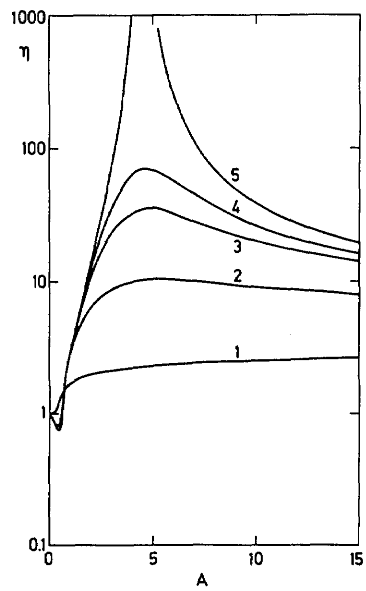

Fig. 2. Enhancement factor η as a function of the dimensionless pump intensity A for various values of the dimensionless frequency detuning Δ: 1, $\Delta = 0$; 2, $\Delta = -\sqrt{3}(1-0.2)$; 3, $\Delta = -\sqrt{3}\,(1-2/35)$; 4, $\Delta = -\sqrt{3}\,(1-1/35)$; 5, $\Delta = -\sqrt{3}$.

$$A_{max} = -2(\Delta^2 + 1)[(\Delta^2 + 1)^{1/2} + 1]^2/3\Delta^3. \quad (10)$$

It is interesting to note that, even far from the instability threshold, the enhancement remains large. In the limit $A \rightarrow \infty$, $\eta \rightarrow 3$, independently of Δ. We also remark that the enhancement is only weakly sensitive to changes of the driving field (see, for instance, curve 3 of Fig. 2), which indicates the possibility of using pump lasers with relatively high-intensity fluctuations.[5]

We illustrate our results with the following example. We assume large mirror reflectivity ($T \cong 0.1$), neglect the internal losses ($\gamma_s \ll \gamma_c$), and take advantage[6] of the large $\chi^{(3)}$ in InSb [$\chi^{(3)} = 10^{-2}$ esu]. Considering $L_s \cong 100\ \mu$m and $k_0 \cong 1.2 \times 10^4$ cm^{-1}, we obtain, with the threshold condition $A_{th} = 8/\sqrt{3}$, $|E_I|_{th} \cong 45$ V/cm. For a beam area of 1 mm × 1 mm, this corresponds to an incident power of about 50 mW, which can readily be achieved in the cw regime.

This enhancement of the Sagnac effect could obviously find applications in cases in which high sensitivies are required (e.g., optical tests of general relativity). On the other hand, this effect provides a direct way to study experimentally nonlocal interactions of light with matter, since it is due solely to the formation of a nonlinear index grating, which in turn can be formed if the nonlinearity depends only on the light intensity at a given point (local interaction). If the index grating is washed out by some fast process (for instance, diffusion in gases), the nonreciprocal factor 2 in Eq. (5) is replaced with $1 + \alpha$, where in the general case $0 \leqq \alpha \leqq 1$. This leads to a reduced enhancement factor $\eta_\alpha = 1 + \alpha(\eta - 1)$. Thus the measurement of η_α provides a novel spectroscopic method to analyze nonlocal interactions (nonreciprocal Sagnac spectroscopy).

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