DIRECTIONALLY ASYMMETRICAL BISTABILITY IN A SYMMETRICALLY PUMPED NONLINEAR RING INTERFEROMETER

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We consider a nonlinear ring resonator pumped symmetrically by two beams of equal intensities and opposite directions. We show that this system is characterized by a new directionally asymmetrical regime of multistability. This is due to the non-reciprocity of propagation of the counterpropagating waves in the resonator produced by a nonlinear index grating.

Optical bistability has been the object of intense activity over the last few years [1]. Up to now, the bulk of the research has been concerned with nonlinear Fabry-Perot and ring cavities pumped by a single field [2]. In this letter, we consider the situation of a nonlinear ring resonator *symmetrically* pumped by two beams of the same intensity and opposite directions (see fig. 1). We show that beside the expected usual bistability, this system is characterized by a new instability leading to a *non-symmetrical* regime, i.e., the amplitudes of the two counterpropagating waves become different. This new feature is due to the cross-interaction of the two-waves via the nonlinear index grating that they generate. We immediately note that this effect could not occur in the case of single-wave pumping [3]. Furthermore, it disappears in situations where the grating is washed out by some physical process (e.g., diffusion in vapors), even if the medium still exhibits a strong nonlinearity.

The existence of an asymmetric regime is of great interest for applications where directional optical switching is required. Furthermore, we have shown [4] that it provides us with a way to enhance the Sagnac effect by several orders of magnitude. We note that asymmetric optical bistability, based on the use of two-photon processes [5] or 3-level atoms [5, 6] as a nonlinear medium, has previously been proposed. Our system presents the advantage of not requiring a resonant medium, and not using an absorption mechanism.

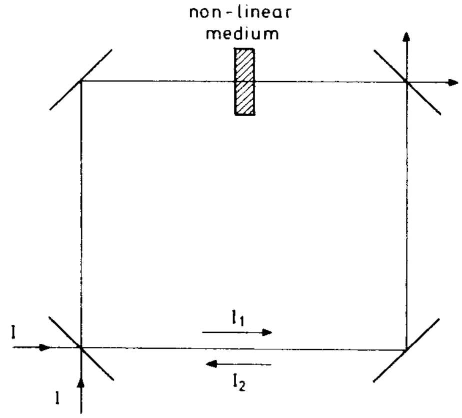

Fig. 1. Ring resonator with nonlinear elements and pump by two beams of the same intensity.

We consider the case of a Kerr-like nonlinear medium, which has a nonlinear susceptibility of the form $\epsilon = \epsilon_0 + \Delta\epsilon^{NL}$, where $\Delta\epsilon^{NL} = \epsilon_2|E(x)|^2$. Here, $E(x) = E_1(x)e^{ikx} + E_2(x)e^{-ikx}$ is the total field at location x, and E_1 and E_2 are the amplitude of the clockwise and anticlockwise propagating field, respectively.

It is well-known [7,4] that in the presence of two counter-propagating waves, the nonlinear component $\Delta\epsilon^{NL}$ is different for opposite directions, namely

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$$\Delta\epsilon_1^{NL} = \epsilon_2(|E_1|^2 + 2|E_2|^2) ;$$

$$\Delta\epsilon_2^{NL} = \epsilon_2(|E_2|^2 + 2|E_1|^2) . \quad (1)$$

The asymmetrical factors in eq. (1) result from the formation of a nonlinear index grating which leads to the cross interaction of E_1 and E_2. One can easily show that for a high finesse ring resonator the steady-state amplitude of the field E_j is given by [4]

$$E_j = \frac{(\gamma_c/\gamma)E_I/\sqrt{T}}{1 + i\gamma^{-1}[\nu - \nu_0 + \nu_0\Delta\epsilon_j^{NL}L_s/L]} . \quad (2)$$

Here, E_I is the incident field, T is the mirror transmittivity, L the total optical length of the resonator and L_s the optical length of the nonlinear sample, $\gamma_c = cT/L$ is the empty cavity bandwidth, and γ_s gives the linear losses of the medium inside the cavity, so that $\gamma = \gamma_c + \gamma_s$ gives the total bandwidth of the system, and $k_0 = \omega_0/c$. The good finesse condition is $\gamma \ll c/L$. We introduce the dimensionless detuning $\Delta = (\upsilon - \upsilon_0)/\gamma$ and the dimensionless intensities

$$I_j = \epsilon_2|E_j|^2L_s\nu_0/L\gamma , \quad A = [\epsilon_2|E_I|^2L_s\nu_0/L\gamma T](\gamma_c/\gamma)^2 .$$

The equation for the intensity I_j is readily obtained by multiplying eq. (2) by its complex conjugate:

$$A = I_j\{1 + [\Delta + I_j + 2I_{3-j}]^2\} . \quad (3)$$

We consider two different solutions, the symmetrical and asymmetrical ones. In the symmetrical case, ($I_1 = I_2 = I_0$), the intensities satisfy the equation

$$A = I_0\{1 + (\Delta + 3I_0)^2\} . \quad (4)$$

The intensity I_0 as function of the incident intensity A is shown in fig. 2 (symmetrical branch) for various values of the detuning Δ. As can be seen from figs. 2b, c, d, the symmetrical regime exhibits hysteresis for sufficiently large values of Δ and A. By requiring that $dI_0/dA = \infty$ at the points of hysteretic jump, we find readily from eq. (4) that the domain of symmetrical bistability is given in the plane of the parameters A and Δ by the relation

$$A = (2/81)[-\Delta(\Delta^2 + 9) \pm (\Delta^2 - 3)^{3/2}] . \quad (5)$$

This domain is shown in fig. 3 as the curve A_2. The

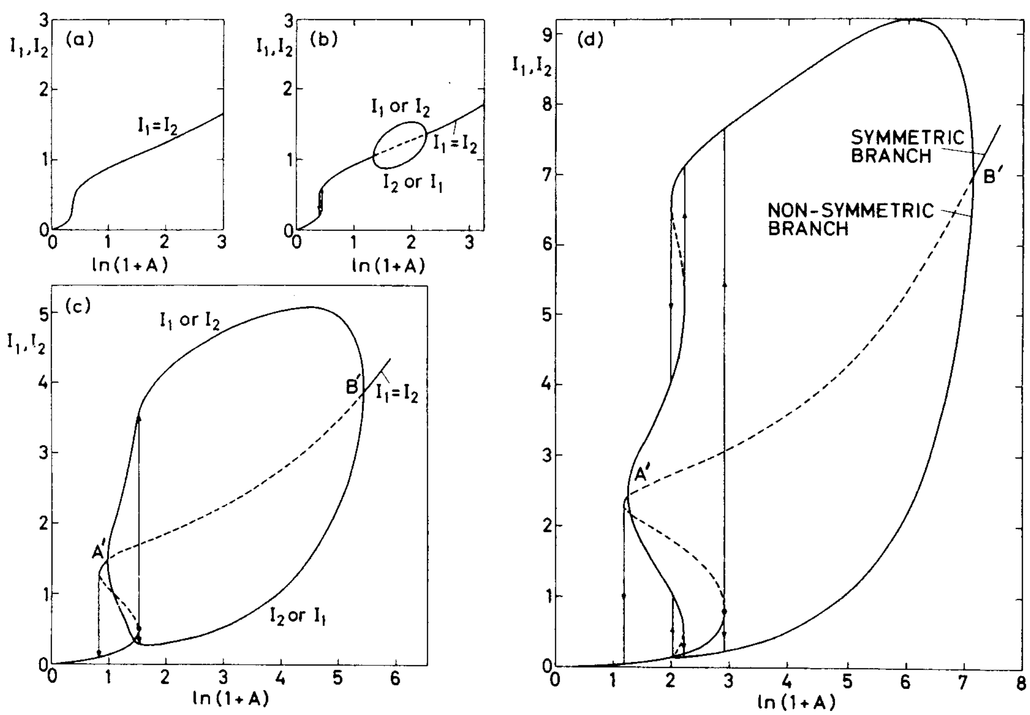

Fig. 2. Normalized intensities I_1 and I_2 as a function of the dimensionless pump intensity A. The dashed parts of the curves indicate the regions of instability. The curves labeled by $I_1 = I_2$ give the symmetrical regime, and the points A′ and B′ the crossings between symmetrical and asymmetrical regimes. The four figures are for various values of the dimensionless detuning Δ. (a) $\Delta = -1.65$; (b) $\Delta = -1.8$; (c) $\Delta = -4$; (d) $\Delta = -7$.

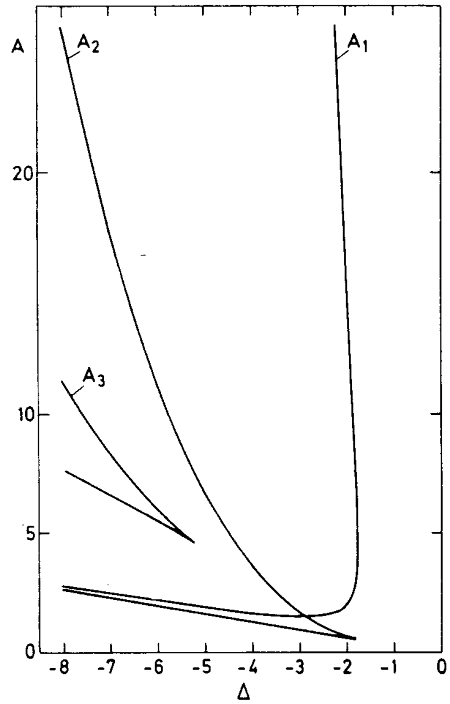

Fig. 3. Domains of operation in the plane of the parameters A and Δ. The curve A_1 defines the domain of instability of symmetrical solution, A_2 the domain of bistability of symmetrical solution and A_3 the domain of multistability of the asymmetrical solution.

detuning threshold for the onset of symmetrical bistability is given by $\Delta_2 = -\sqrt{3}$, which corresponds to a driving intensity $A = 8/9\sqrt{3}$. However, it can easily be shown by a linear stability analysis of eq. (3) that under appropriate conditions, the symmetrical regime determined by eq. (4) becomes unstable. The domain of instability of the symmetrical solution is given by

$$A = (2/3)[-\Delta(3\Delta^2 - 5) \pm (3\Delta^2 - 1)\sqrt{\Delta^2 - 3}] . \quad (6)$$

It is shown as the curve A_1 in fig. 3. The detuning threshold for this regime is $\Delta_1 = -\sqrt{3}$ which is equal to Δ_2, but corresponds to the larger pump intensity $A = 8/\sqrt{3}$. Inside this domain, the stable behaviour of the system is characterized by the fact that I_1 and I_2 are unequal. For $I_1 \neq I_2$, the two coupled third-order equations (3) can be reduced to a single third-order equation for $S = I_1 + I_2$,

$$A = [1 + (S + \Delta)^2]\,[2\Delta + 3S] , \quad (7)$$

which can be solved readily for any values of A and Δ. The separate values of I_1 and I_2 can then be obtained via the equation

$$A = P(2\Delta + 3S) , \quad (8)$$

where $P = I_1 I_2$.

The asymmetric solution is shown in figs. 2 (b, c, d). Note that I_1 and I_2 play completely symmetrical roles in eqs. (7) and (8), so that which of them will become larger is determined by noise or slight initial asymmetries in pumping.

It is interesting to point out that the asymmetrical regime can itself become multivalued for large enough Δ and A, as seen in fig. 2d. Applying the same criteria as in the symmetrical case ($\mathrm{d}I_j/\mathrm{d}A = \infty, j = 1, 2$) we find that the domain of multistability of the asymmetrical branch is given by

$$A = (2/3^5)[-\Delta(\Delta^2 + 81) \pm (\Delta^2 - 27)^{3/2}] . \quad (9)$$

The corresponding threshold of detuning is $\Delta_3 = -3\sqrt{3}$, which gives $A = 8/\sqrt{3}$.

Let us now discuss in detail the behaviour of the system as one increases and decreases the pump intensity. We consider first the case of fig. 2b, which exhibits two separated regions of bistability. As one increases A, the system, initially in the symmetrical regime, first jumps to the high symmetrical branch, exhibiting a bistability which is similar to that occuring in usual Fabry-Perot resonators. As A is increased further, the symmetric regime becomes unstable, and I_1 (or I_2) becomes larger, while I_2 (or I_1) decreases. This asymmetric regime holds until A is so large that the system returns continuously to the symmetrical regime of operation.

If one now decreases A, one first goes again into the asymmetrical regime. For smaller A, the system returns continuously into the upper branch of the symmetrical regime, and eventually jumps back down to the lower symmetric branch.

The case shown in fig. 2c is similar, except that the first jump (for increasing driving field) brings the system directly into the asymmetrical regime. Thus, in this case, one has two overlapping bistable regimes (one symmetrical and the other asymmetrical), giving tristability in the region of overlap.

We now turn to the case illustrated in fig. 2d. The major new feature, as compared to the previous case, is that the system becomes pentastable (5 stable states, one of them being symmetrical while the others are asymmetrical). Thus, when decreasing the pump intensity from a very high value, the system exhibits a discontinuous jump in the asymmetrical regime. If one increases A again, one covers an hysteric cycle in this regime.

We note that the effects discussed in this paper can be obtained readily with relatively small intensities. The threshold intensity for the appearance of asymmetrical bistability is $A_{th} = 8/\sqrt{3}$. If we consider large mirror transmittivity ($T \approx 0.1$), neglect the internal losses ($\gamma_s \ll \gamma_c$), and take advantage [8] of the large $\chi^{(3)}$ in InSb ($\chi^{(3)} = 10^{-2}$ esu), take $L_s \approx 100\ \mu$m and $k_0 \approx 1.2 \times 10^4\ \mathrm{cm}^{-1}$, we obtain $|E_I|_{th} \approx 45$ V/cm. This corresponds to an incident power of about 50 mW/mm^2 which can readily be achieved in the cw regime.

Finally, we remark that in the asymmetrical regime, the switch from one dominant direction of propagation to the opposite one could be achieved by changing the intensity of one of the pump beams, the other being kept constant.

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References

[1] For recent comprehensive reviews, see Optical bistability, eds. C.M. Bowden, M. Ciftan and H.R. Robl (Plenum, 1981), and special issue of IEEE J. Quantum Electronics on Optical Bistability, QE-17 (March 1981). For an introduction to the subject, see P.W. Smith and W.J. Tomlinson, IEEE Spectrum 18 (1981) 26.

[2] Optical bistability at nonlinear interfaces (without cavity) has been studied extensively by A. Kaplan, in: Optical bistability, op. cit. p. 447; P.W. Smith and W.I. Tomlinson, Optical bistability, op. cit. p. 463, and references therein.

[3] See for instance R. Bonifacio, R. Gronchi and L. Lugiato, Optical bistability, op. cit. p. 31; J.D. Cresser and P. Meystre, Optical bistability, op. cit. p. 265, and references therein.

[4] A.E. Kaplan and P. Meystre, Optics Lett., to be published.

[5] D.F. Walls, C.V. Kunasz, P.D. Drummond and P. Zoller, Phys. Rev. A24 (1981) 627.

[6] M. Kitano, T. Yabuzaki and T. Ogawa, Phys. Rev. Letters 46 (1981) 926.

[7] B.Ya. Zeldovich, Brief Communications in Physics of the Lebedev Institute No. 5 (1970) 20 (in Russian); M. Sargent III, Appl. Phys. 8 (1976) 127.

[8] D.A.B. Miller, S.D. Smith and B.S. Wherrett, Optics Comm. 35 (1980) 221.