description, suggesting that a modified framework must be used to provide a proper description of transition-metal solids. Since a key aspect of the bonding involves the U_{ij} or $S_i \cdot S_j$ terms, perhaps the local-density formulations can be modified to explicitly include such terms (e.g., $\sum_{i,j} J_{ij} S_i \cdot S_j$) where the $J_{ij}(R)$ are obtained from molecular calculations.

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Hysteresis in Cyclotron Resonance Based on Weak Relativistic-Mass Effects of the Electron

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The cyclotron resonance is considered upon action of strong electromagnetic quasiresonant wave. It is shown that even a very weak relativistic ($\beta^2 \ll 1$) mass effect of the electron can result in large hysteretic jumps of its steady-state kinetic energy, if the wave intensity or frequency is varied.

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This paper is basically to attract attention to the fact that even a very weak relativistic ($\beta^2 \ll 1$) mass effect can result in large nonlinear effects in such a very well studied phenomenon as free-electron cyclotron resonance. The proposed effect is important because it suggests for the first time, to our best knowledge, bistable interaction of an electromagnetic wave with the simplest microscopic physical object. This differs fundamentally from all kinds of optical bistability[1-3] presently known, which so far has always been based on macroscopic nonlinear properties of the media. Nonlinear change in macroscopic susceptibility under action of the strong EM wave provides dramatic change in the optical condition of propagation of this wave under various special circumstances (e.g., in nonlinear Fabry-Perot resonators[1]; at nonlinear interfaces[2]; or in counterpropagating beams of light,[3] interacting with each other) which, in turn, leads again to the change in the susceptibility. This so-called optical feedback in nonlinear macrosystems results in the existence of multistable (in particular, bistable) steady states. No such optical feedback exists in the case considered in this paper.

The proposed effect is based on the dependence of the cyclotron frequency of forced oscillation on the intensity of the incident driving wave; this dependence is due to the relativistic change of the mass of electron. If the relativistic shift of cyclotron frequency becomes sufficiently larger than the frequency width of resonance, it dramatically changes the resonant conditions of excitation of an electron. That can result, in turn, in large hysteretic jumps. In essence, this effect is analogous to the hysteresis in a classical nonlinear oscillator,[4] or in nonlinear parametrical systems.[5]

To illustrate feasibility of this effect, we shall demonstrate it for the simplest case of a single electron immersed in a very strong constant magnetic field H_0, and interacting with a strong electromagnetic field $E_{\rm in}$ of amplitude E (such that $E \ll H_0$). The EM field propagates along the axis z parallel to H_0. Field H_0 provides a cyclotron resonance with the initial frequency ω_0 eH_0/m_0c where e is electrical charge of the electron, m_0 is its rest mass, and c is the speed of light. We also assume that a small constant electric field $\vec{g}(z)$ is applied along axis z, which is provided by some potential to arrange a trapping of the electron[6] and to compensate a radiation force caused by the EM wave.

We treat this problem classically. The equation of motion for the electron moving with arbitrary velocity is[7]

$$d(m\vec{v})/dt=(e/c)\vec{v}\times\vec{H}_\Sigma+e\vec{E}_\Sigma+\vec{F}_l,\qquad m=m_0(1-|\vec{v}|^2/c^2)^{-1/2}, \tag{1}$$

where $\vec{H}_\Sigma$ is the total magnetic field (including the EM wave component), $\vec{E}_\Sigma=\vec{E}_{\rm in}+g(z)\hat{e}_z$ is the total electric field, and the term $\vec{F}_l$ represents energy losses of the electron. In the ultimate case in which the losses are caused by EM radiation of the rotating electron (and $|v|\ll c$) this term can be written[7] as $\vec{F}_l=(2e^2/3c^2)d^2\vec{v}/dt^2$. In the general case the losses are much larger and are caused by various factors, basically by collisions, so that the radiation losses could be neglected. The force is then proportional to the velocity of the electron, e.g., $\vec{F}_l=-\Gamma m_0\omega_0\vec{v}$, where Γ is the dimensionless width of cyclotron resonance. The radiation losses can also be represented by this formula, since one can assume[7] that $d^2\vec{v}/dt^2\approx-\omega_0^2\vec{v}$, which yields

$$\Gamma_{\rm rad}=\frac{2e^2\omega_0}{3m_0c^3}=\frac{2}{3}r_ek_0\ll 1,$$

where $r_e=e^2/m_0c^2=2.8\times10^{-13}$ cm is an electron radius and $k_0=\omega_0/c$ is a resonance wave number. Let us introduce dimensionless notations:

$$\vec{\beta}=\vec{v}/c,\quad \vec{\mu}=\vec{E}/H_0,\quad g(z)/H_0=\rho(z). \tag{2}$$

All these parameters are supposed to be small compared to unity. We see below that near the bistable regime, their orders of magnitude are related to each other by the relations $\beta\sim\mu/\Gamma$; $\beta^2\sim\rho/\Gamma\sim\Gamma$. This yields an even stronger hierarchy of parameters:

$$1\gg|\vec{\beta}|\gg\Gamma\gg|\vec{\mu}|\gg\rho. \tag{3}$$

Using these conditions, the definitions (2), and the conventional Maxwell's equations for computing the wave component of the total magnetic field, we rewrite Eq. (1) in the form

$$\omega_0^{-1}\left(1+\frac{|\vec{\beta}|^2}{3}\right)\frac{d\vec{\beta}}{dt}=\vec{\mu}+\vec{\beta}\times\hat{e}_z+\hat{e}_z[\vec{\beta}\cdot\vec{\mu}-\rho(z)]-\Gamma\vec{\beta}. \tag{4}$$

Here $|\vec{\beta}|^2/2$ represents a small nonlinearity which is due to weak-relativistic mass effect, and the term $\vec{\beta}\cdot\vec{\mu}$ is the radiation pressure of the incident wave. Let us now assume that this wave is circularly polarized,[8] and rotates in the same direction as the electron:

$$\vec{\mu}=\mu[\hat{e}_x\sin(\omega t-kz)+\hat{e}_y\cos(\omega t-kz)],\qquad k=\omega/c, \tag{5}$$

where ω is a frequency of the field. The required solution to (4) can then be written in the form

$$\vec{\beta}(t,z)=\beta[\hat{e}_x\sin(\omega t+\varphi)+\hat{e}_y\cos(\omega t+\varphi)]+\beta_z\hat{e}_z. \tag{6}$$

According to (3), the unknown values β, β_z, and φ vary little in the time $1/\omega$, which allows us to neglect their second-order time derivatives and higher harmonics, and to write down the set of truncated first-order equations

$$\dot{\beta}/\omega_0=-\Gamma\beta+\mu\cos(\varphi+kz),$$

$$\dot{\varphi}/\omega_0=-\left[\Delta+\frac{\beta^2}{2}+\frac{\mu}{\beta}\sin(\varphi+kz)\right], \tag{7}$$

$$\dot{\beta}_z/\omega_0=-\rho(z)+\mu\beta\cos(\varphi+kz),\quad \dot{z}=c\beta_z,$$

where $\Delta=(\omega-\omega_0)/\omega_0\ll1$; Δ is the dimensionless resonant detuning. The steady-state solution ($d/dt=0$) is thus determined by the relationships

$$\mu^2=\beta_s^2[\Gamma^2+(\Delta+\beta_s^2/2)^2],\qquad \beta z_s=0,\quad \rho(z_s)=\Gamma\beta_s^2, \tag{8}$$

$$\tan(\varphi_s+kz_s)=-\Gamma^{-1}(\Delta+\beta_s^2/2), \tag{9}$$

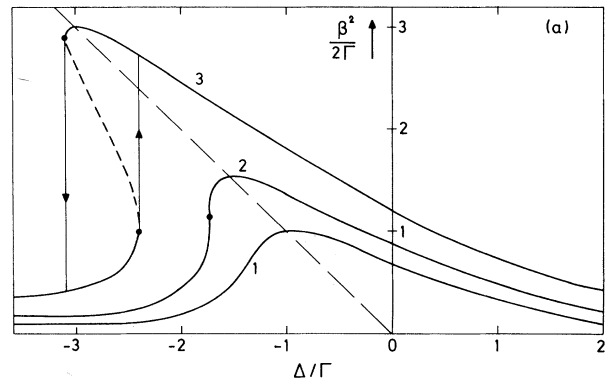
where the subscript "s" labels characteristics of the steady-state regime. It is seen from Eq. (8) that the steady-state kinetic energy $\beta^2/2$ does not depend on parameters of the force $\rho(z)$. This force serves a secondary role canceling the radiation pressure of the incident wave and is very small compared to all the other forces in the problem; e.g., in the region of interest $\rho\sim\Gamma^2\sim\beta_s^4$. [The force $\rho(z)$ can be provided by the trapping potential in a Penning trap[6] (in vacuum) or by the background charge (in semiconductors or plasmas).] It is obvious, though, that characteristics of the transient regime, described by Eq. (7), depend on the spatial behavior of the force $\rho(z)$. For example, the frequency of transient oscillations along the z axis should depend upon $d\rho/dz$ in the vicinity of $z=z_s$, Eq. (9). However, the magnitude of these oscillations at the onset of the hysteretic jumps can be assumed to be arbitrarily small, provided that the amplitude (or the frequency) of the incident wave is scanned sufficiently slowly to prevent the transient regime from masking the hysteretic behavior of the steady-state regime.

It can be easily seen that under the threshold conditions

$$\mu^2>\mu_{th}^2\equiv(16/3\sqrt{3})\Gamma^3,\quad \Delta<\Delta_{th}\equiv-\Gamma\sqrt{3}, \tag{10}$$

Eq. (8) yields a three-valued solution for β_s. The plot of steady-state kinetic energy $\beta_s^2/2$ as a function of resonant detuning Δ and incident intensity μ^2 is shown in Fig. 1. At the threshold point this value is $\beta_{th}^2/2=2\Gamma/\sqrt{3}$ (curves 2 in Fig. 1), and the radius of orbit is $\Gamma=\beta_{th}/k_0\ll\lambda_0$. In the case of the multivalued solution the study of Eqs. (7), linearized in the close vicinity of the steady-state solutions (8) and (9), shows that only those states are stable which satisfy the energy criterion $d(\beta_s^2)/d(\mu^2)>0$ (solid branches of the curves in Fig. 1); otherwise, they become unstable (dashed branches in Fig. 1). That meets the physical expectations, and leads to hysteretic behavior of the electron under conditions (10).

Let us make some quantitative estimates. A magnetic field of strength $H_0=100$ kG produces the cyclotron frequency $\omega_0=1.7\times10^{12}$ sec^{-1} ($\lambda_0\sim1.07$ mm). Then in the ultimate case of radiation losses, the resonance width is $\Gamma\sim10^{-11}$, which yields the threshold field amplitude to be as small as $E_{th}=1.7\times10^{-9}$ V/cm, and the kinetic energy as small as $\beta_s^2/2\simeq1.2\times10^{-11}$. It is, in fact, even near α^{-1} times smaller than a quantum limit of the energy of excitation which is $2\hbar\omega_0/mc^2$ (here $\alpha=e^2/\hbar c=1/137$ is the fine-structure constant). In more realistic cases, the losses are caused by any interaction of the electron with other particles and a resonator. Assuming the losses to be, for instance, $\Gamma\simeq10^{-4}$, one still has the easily obtainable threshold field $E_{th}\simeq54$ V/cm. That corresponds to the incident power of about 70 mW/mm^2 which can readily be achieved in cw regime. The kinetic energy of the electron at the onset of hysteresis is then $\beta_{th}^2/2\simeq1.2\times10^{-4}$, which corresponds to $mv^2/2\simeq60$ eV.



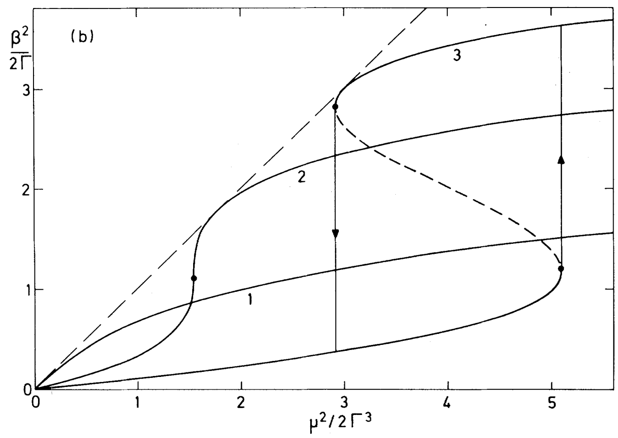


FIG. 1. The plots of normalized kinetic energy of the electron $\beta^2/2$ (a) vs normalized resonant detuning Δ/Γ for various intensities of incident EM field, and (b) vs normalized incident intensity $\mu^2/2\Gamma^3$ for various detunings. Curves: (a) 1, $\mu^2/2\Gamma^3=1$; 2, $\mu^2/2\Gamma^3=(2\sqrt{3})^3$; 3, $\mu^2/2\Gamma^3=3$; (b) 1, $\Delta/\Gamma=0$; 2, $\Delta/\Gamma=-\sqrt{3}$; 3, $\Delta/\Gamma=-3$.

It is obvious that the effect discussed here can be expected also in the case of cyclotron resonance in solid-state materials. It is well known[9] that the effective mass of the electron in some semiconductors (e.g., InSb) is strongly dependent on energy of its excitation, which should cause some shift of cyclotron frequency under action of the strong resonant EM pumping. This effect, being in some sense analogous to the relativistic mass effect, can result in strong hysteresis, if the nonlinear shift of the cyclotron frequency is larger than frequency width of resonant line. The effective mass of the electron m^* in InSb could be as small as $m^* \approx 0.02\ m_0$, which for the case of $H_0 = 100$ kG produces the cyclotron wavelength $\lambda_0 \sim 20\ \mu$m, so that cw radiation of the CO_2 laser will be suitable to excite hysteretic cyclotron resonance.

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Variable-Wiggler Free-Electron-Laser Experiment

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First results are presented of a free-electron-laser experiment that utilizes a tapered wiggler for efficiency enhancement. The spontaneous spectrum and the electron-beam energy loss due to the free-electron-laser interaction are measured and compared with theory. With a 2.25% magnetic field taper, 12% of the electrons were decelerated by 0.6%, corresponding to a gain of 2.7% and an efficiency of 0.07%, which is 10 times higher than that calculated for a zero taper and otherwise identical conditions.

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An efficient free-electron laser (FEL) is a viable source of high-power radiation in the visible regime. It has been shown[1,2] theoretically that the FEL efficiency, inherently low at small wavelengths ($\lambda_s \leq 10.6\ \mu$m), can be increased by appropriately tapering the wiggler field. In this paper, we present the encouraging initial results of the TRW FEL amplifier experiment being performed to prove the validity of this scheme.

A FEL[3] generates stimulated radiation by the interaction of a relativistic electron beam with a rippled magnetic field (wiggler). The wavelength,

