Optoelectronic enhancement of the Sagnac effect in a ring resonator and related effect of directional bistability

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A substantial enhancement (by orders of magnitude) of the Sagnac effect in a passive ring resonator can be attained by using a nonreciprocal feedback. This feedback is based on the nonreciprocal element controlled by the signal proportional to the difference between intensities of counterpropagating waves, and is an optoelectronic analog of nonlinear nonreciprocity proposed by us earlier. Under some critical conditions, this system can exhibit directional bistability and directional switching of the counterpropagating waves.

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The Sagnac effect[1] is an underlying phenomenon for optical rotational sensors (laser gyroscopes as well as passive interferometers and ring resonators) which provide an opportunity to measure extremely small rotation rates[2] and could be applied for geophysical research, intertial navigation systems, and related techniques.[3] In our previous work,[4] a novel way to dramatically enhance the Sagnac effect was proposed which is based on nonlinearly induced nonreciprocity. (Under some conditions the nonlinear nonreciprocity can result also in directional bistability.[5])

In this letter we propose an optoelectronic analog of the nonlinear system[4,5] which imitates nonlinear nonreciprocity by using the signal proportional to the difference between intensities of counterpropagating waves in the ring resonant (see Fig. 1) to control the nonreciprocal element (e.g., Faraday rotator[6]) which is incorporated in the resonator and causes a differential change of total optical path lengths of both of the waves. The Faraday rotator may comprise, e.g. of a Faraday cell or Faraday mirror[6] (which might be even preferable because it does not disturb a wave propagation inside the ring resonators) driven by an external magnetic field.

We consider a passive ring resonator (Fig. 1) which is pumped in opposite directions by the laser beams having the same intensity $|E_{\text{in}}|^2$ and frequency ν tuned in such a way that it is close to one of the eigenfrequencies ν_0 of the resonator. Then, in the presence of rotation, the Sagnac effect causes an initial differential change of the optical paths of the waves. In turn, this results in differential changes of the intensities of both of the waves because of the splitting of the eigenfrequencies seen by both beams (one of the beams moves toward a resonance while another moves away from it). This small intensity difference can be detected and amplified by differential amplifier. The output signal of this amplifier is used to drive the nonreciprocal element (e.g., via controlled magnet field in the Faraday cell) which provides further splitting of the eigenfrequencies and further increase of the difference between the intensities. In a way, such a system is an optoelectronic analog of nonlinear-optic passive gyroscope proposed in Refs. 4 and 5, i.e., nonreciprocal feedback could be considered as "artificial" nonlinear nonreciprocity. Another system, based on the same idea of nonreciprocal feedback, was recently proposed[7] by us for a laser gyroscope in which case the feedback signal is proportional directly to the frequency splitting of oscillations in laser gyro.

Suppose that the nonreciprocal response (e.g., the nonreciprocal shift of each eigenfrequency, $\Delta\nu_{\text{nr}}$) provided by the feedback is proportional to the difference between the partial intensities of the waves:

$$\Delta\nu_{\text{nr}}/\nu_0 = \alpha(|E_1|^2 - |E_2|^2), \tag{1}$$

where $\alpha = \text{const}$ is a coefficient determined by the parameters of the amplifier, the Faraday cell, etc. (We assume as well, that the detecting and amplifying system can distinguish the sign of difference between the intensities.) Then the steady-state amplitudes $E_j (j = 1,2)$ of each of the two waves transmitted through the output semitransparent mirror are

$$E_j = \frac{E_{\text{in}}\sqrt{\gamma_c/\gamma}}{1 + i\gamma^{-1}\{\nu - \nu_0 + (-1)^j[\omega_s + \alpha\nu_0(|E_1|^2 - |E_2|^2)]\}}. \tag{2}$$

Here E_{in} is an amplitude of the incident (pumping) field, γ_c is the empty cavity bandwidth (i.e., that caused by mirrors only), γ is the total bandwidth of the system ($\gamma = \gamma_c + \gamma_m$, where γ_m gives the linear losses of medium inside the cavity), and

$$\omega_s = 4S\omega_r\nu_0/Lc \tag{3}$$

is the "Sagnac frequency splitting," where ω_r is the rotational rate, S is the area of the ring resonator, and L is its total

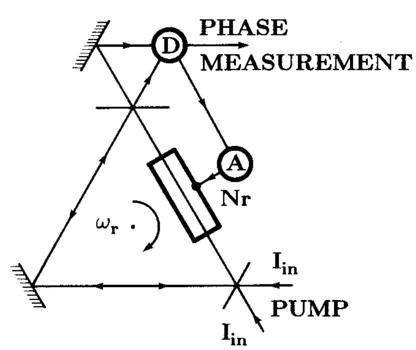

FIG. 1. Schematic of ring resonator gyroscope using an imitation of nonlinear nonreciprocity for substantially enhancing the Sagnac effect; the resonator is pumped in both directions by laser beams with equal intensity and frequency. Nr—nonreciprocal element which may comprise, e.g., of a Faraday cell or a Faraday mirror driven by an external magnet field; D—detecting system providing measurement of phase difference between both of the beams; A—differential amplifier. The signal proportional to the intensity difference controls the nonreciprocal element.

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optic path length. Note that Eq. (2) is an approximate form valid for the good finesse case, that is $\gamma \ll c/L$. We introduce the dimensionless resonance detuning $\Delta = (\nu - \nu_0)/\gamma$ and the dimensionless rotational rate $\Omega = \omega_s/\gamma$, as well as the dimensionless intensities:

$$I_j = \alpha\nu_0|E_j|^2/\gamma; \quad (j = 1,2); \quad I_{\text{in}} = \alpha\nu_0\gamma_c|E_{\text{in}}|^2/\gamma^2. \qquad (4)$$

(Note that I_j and I_{in} change sign if $\alpha < 0$.) The equations for the intensities I_j are readily obtained by multiplying Eq. (3) by its complex conjugate:

$$I_{\text{in}} = I_1[1 + (\Delta - \Omega - I_1 + I_2)^2] = I_2[1 + (\Delta + \Omega + I_1 - I_2)^2]. \qquad (5)$$

We assume that in the absence of rotation ($\Omega = 0$), the intensities I_1 and I_2 are equal:

$$I_1 = I_2 = I_0 \equiv I_{\text{in}}/(1 + \Delta^2). \qquad (6)$$

Then, in the most interesting case of small rotations ($\Omega \ll 1$), one can assume that the perturbations $\zeta_j = I_j - I_0$ are small ($\zeta_j \ll I_0$). Under this condition, Eq. (5) immediately yields for ζ_j:

$$\zeta_1 = -\zeta_2 = \Omega(\eta - 1)/2, \qquad (7)$$

where

$$\eta = [1 - 4I_{\text{in}}\Delta/(1 + \Delta^2)^2]^{-1}. \qquad (8)$$

Without any nonreciprocal feedback, the Sagnac effect is determined by the term ω_s in Eq. (2) with $\alpha = 0$. It can be seen now from Eqs. (5) and (7) that with the feedback, the total phase difference between the two counterpropagating waves is determined by $\eta\omega_s$ instead of by ω_s. Hence, the result of the nonreciprocal feedback is to scale the Sagnac effect by a factor of η which is to be considered as an enhancement factor. Its behavior as a function of the resonant detuning Δ for various pumping intensities I_{in} is shown in Fig. 2. If $I_{\text{in}} < 4/3\sqrt{3}$, the maximum and minimum of η, namely, $\eta_m = (1 \pm 3I_{\text{in}}\sqrt{3/4})^{-1}$, occur at $\Delta = \pm 1/\sqrt{3}$. It is seen from Eq. (8) that at the boundary

$$I_{\text{in}} = I_{\text{th}} \equiv (1 + \Delta^2)^2/4\Delta \qquad (9)$$

enhancement becomes infinitely large ($\eta \to \infty$), i.e., Eq. (9) defines the boundary of the domain of instability (domain 1 at Fig. 3). The enhancement increases dramatically as one approaches the boundary (9) from the outside of this domain (i.e., $0 < I_{\text{th}} - I_{\text{in}} \ll I_{\text{th}}$). Let us make quantitive estimates. We

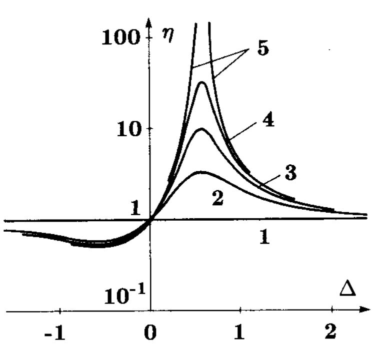

FIG. 2. Enhancement factor η vs the dimensionless resonant detuning Δ for various pumping intensity I_{in}. Curves 1: $I_{\text{in}} = 0$; 2: $I_{\text{in}}/I_{\text{cr}} = 0.7$ (where $I_{\text{cr}} = 4/3\sqrt{3}$); 3: $I_{\text{in}}/I_{\text{cr}} = 0.9$; 4: $I_{\text{in}}/I_{\text{cr}} = 0.97$; 5: $I_{\text{in}} = I_{\text{cr}}$.

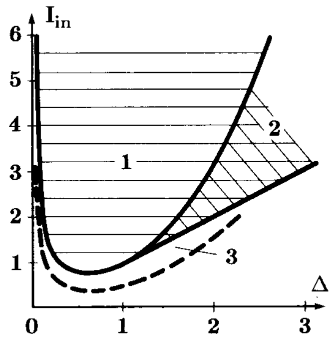

FIG. 3. Domains of operation in the plane of parameters I_{in} and Δ: 1—domain of instability of the symmetric solution $I_1 = I_2$ and existence of directional bistability (with the asymmetric solution $I_1 \neq I_2$); 2—domain of multistable solution; 3—domain of significant ($\eta > 2$) enhancing the Sagnac effect.

assume that stability of the pumping intensity $|E_{\text{in}}|^2$ (as well as that of the feedback circuit) is of order of 10^{-4} (e.g., for stabilized argon laser[8]), while the stability of the resonant detuning ($\omega - \omega_0$) is even much better.[8] It can then be seen from Eq. (8) with $\Delta = 1/\sqrt{3}$ that the enhancement of $\eta \sim 1.5 \times 10^2$ with stability of 10^{-2} can readily be attained. The difference between the partial intensities $|E_1|^2 - |E_1|^2$, which is to be monitored by the differential detector, is sufficiently large even for very small rotation rates. We assume that $L = 1$ m, $S \sim 1$ m^2, $\lambda_0 \sim 0.5\,\mu$, $\gamma \sim \gamma_c \sim cT/L$ (T transmission of a mirror; we assume $T \sim 10^{-2}$). Equation (7) then gives $(I_1 - I_2)/I_0 \sim 10^2\eta\omega_r/2\pi$, where ω_r is expressed in s^{-1}. Therefore, if $\eta \sim 100$ and $\omega_r \sim 2\pi \times 10^{-6}$ s^{-1}, then $(I_1 - I_2)/I_0 \sim 10^{-2}$, which can be easily detected and still is by two orders of magnitude larger than the intensity deviations associated with the stability 10^{-4}. For the considered case, the limitation on the lowest measured rotation rate (with enhancement ~ 100) imposed by the stability of intensity 10^{-4} is $\omega_r \simeq 2\pi \times 10^{-8}$ s$^{-1} \simeq 10^{-3}$ earth rotation.

Inside the domain of instability (domain 1 at Fig. 3), any small initial difference between I_1 and I_2 grows up exponentially such that steady state $I_1 = I_2$, Eq. (6), becomes unstable. This results in formation of *asymmetric* oscillation: the amplitude of clockwise wave is different from that of counterclockwise wave, even *in the absence of rotation*, with the sign of this difference determined by noise or initial conditions. Therefore, the stable regimes with $I_1 > I_2$ and $I_1 < I_2$ are equally probable, and could be switched from one into another, which could be considered as a directional bistability. This kind of bistability is again analogous to that studied in Ref. 5.

To find out the characteristics of the asymmetrical regime, one can use again Eqs. (2) and (5) with $\Omega = 0$, looking now for solutions with $I_1 \neq I_2$ [in addition to a symmetrical solution (6)]. Equation (5) readily yields

$$I_{1,2} = \frac{1}{2}\left(\Delta \pm \frac{1}{\Delta}\sqrt{\Delta(I_{\text{in}} - \Delta)} \pm \sqrt{\Delta^2 - 1 \pm 2\sqrt{\Delta(I_{\text{in}} - \Delta)}}\right). \qquad (10)$$

Here, for each set (I_1, I_2), the sign in front of the radicals $\sqrt{\Delta(I_{\text{in}} - \Delta)}$ should be the same; both signs in front of the large radical give one set (I_1, I_2). Behavior of the intensities I_1 and I_2 as function of the pumping intensity I_{in} for fixed detunings Δ is shown in Fig. 4. It is seen that for $\Delta < 0$ (curve 1) the oscillations are always symmetrical ($I_1 = I_2$). For

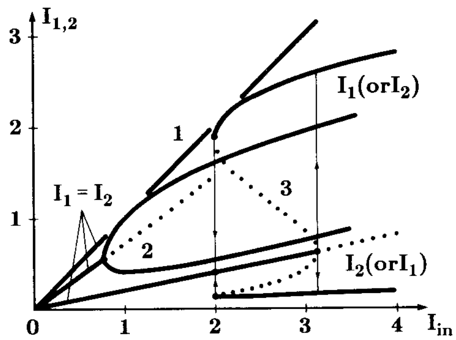

FIG. 4. Dimensionless intensities I_1 and I_2 of the counterpropagating waves vs pumping intensity I_{in} for various resonant detunings Δ. The dotted part of the curves indicates unstable regimes. The curves labeled by $I_1 = I_2$ correspond to the symmetric solution. Curves 1: $\Delta = 0$; 2: $\Delta = 0.6$; 3: $\Delta = 2$.

$0 < \Delta < 1$ (curve 2), one can see that above the threshold ($I_{in} > I_{th}$) intensities of oscillation become different ($I_1 \neq I_2$) which correspond to the *asymmetric* regime or the directional bistability ($I_1 > I_2$ or $I_1 < I_2$). Note that I_1 and I_2 play completely symmetrical roles in Eq. (5), so that which of them becomes larger is determined only by noise or initial conditions.

Furthermore, it follows from Eq. (10) that for $I_{in} > 1$ and $\Delta > 1$, the asymmetrical regime can itself become multivalued (curve 3 at Fig. 4). In such a case, Eq. (10) provides four possible solutions; two of them, which satisfy condition $d(I_1 + I_2)/dI_{in} > 0$, are stable (i.e., those with the upper sign in front of the radicals $\sqrt{\Delta (I_{in} - \Delta)}$), while two others are unstable. This provides an opportunity for the existence of a triple-stable regime (the third state is the symmetric regime $I_1 = I_2$ which remains stable in this case). By requiring that $dI_j/dI_{in} = \infty$ at the border, we find readily that the multivalued domain in the plane of the parameters I_{in} and Δ is defined as $I_{th} > I_{in} > \Delta$, where I_{th} is as given by Eq. (9) (i.e., domain 2 in Fig. 3). Therefore, when increasing the pump intensity I_{in} from a zero value, the system exhibits a discontinuous jump to the asymmetrical regime at $I_{in} = I_{th}$. If now one decreases I_{in}, there is a backward jump into the symmetric regime at $I_{in} = \Delta < I_{th}$, such that one covers an hysteretic cycle.

In conclusion, we propose the use of nonreciprocal optoelectronic feedback to dramatically enhance the Sagnac effect in a passive ring resonator. The feedback is comprised of the nonreciprocal element driven by the signal proportional to the splitting of intensities of counterpropagating waves in the resonator. To some extent, this system provides an optoelectronic analog of the nonlinear enhancement of the Sagnac effect proposed in Ref. 4, and therefore, can be used to model the latter. On the other hand, the proposed system is of considerable interest for gyro applications on its own. This basically is due to the fact that the main disadvantage (which consists in slow relaxation time of the feedback) of the system is not substantially important for the gyroscope because the required response time is usually fairly large.

It is shown that the factor of enhancement η becomes large in the proximity of the threshold of asymmetrical instability of oscillation. An estimate shows that if the stability of the feedback and the pumping intensity is maintained with accuracy of $\sim 10^{-4}$, the enhancement of $\eta \sim 100$ with stability of 1% can be attained and rotation rates as low as 10^{-3} earth rotation can be measured for typical parameters of the resonator. Inside the domain of instability of symmetrical regime, a new, asymmetric regime establishes with dominant mode (clockwise or anticlockwise) determined by initial conditions. This regime corresponds to the directional bistability[5] and therefore could be used for directional switching of radiation.

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