BISTABLE CYCLOTRON RESONANCE IN SEMICONDUCTORS

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We consider the cyclotron resonance in a semiconductor which has a band with a relativistic energy–momentum dispersion and show that, when subject to quasi-resonant radiation, the energy of a free carrier in this band and the relative potential drop between the two surfaces of the semiconductor facing the radiation exhibit bistability under classical single electron approximation.

In a previous work by one of the authors [1], it was shown that cyclotron resonance of relativistic electrons exhibits bistability due to energy dependence of the relativistic cyclotron frequency. In the present letter, we consider the possibility of bistable cyclotron resonance in a semiconductor which has a nonparabolic band with a relativistic momentum dispersion in at least part of the first Brillouin zone [2–4]. By means of a classical analysis, we demonstrate that the cyclotron of free carriers in the semiconductor exhibits bistability if the semiconductor is immersed in a homogeneous magnetic field $\boldsymbol{B}_0$ and irradiated by a circularly polarized light propagating along $\boldsymbol{B}_0$.

The present analysis is similar to that of ref. [1]. However, there are a number of novel features in the present problem. First, the nonlinearity of free carriers in semiconductors are several orders of magnitude larger than the nonlinearity of relativistic electrons in vacuum. This lowers the threshold intensities for the onset of hysteretic jumps in semiconductors, despite rapid thermal relaxation which broadens the cyclotron resonance. In contrast to ref. [1], the strength of the nonlinearity does not allow one to neglect the velocity terms higher than the fourth order and the harmonics higher than the first. We give an exact result in the steady state. Second, effective masses can be quite small in semiconductors. For instance, in narrow gap semiconductors they are nearly two orders of magnitude smaller than the bare electron mass in the vicinity of the band edge [5]. This increases the cyclotron frequency, making it possible to pump carriers with optical or near optical frequencies. For example, if the semiconductor is n-type InSb and $B_0 \sim 140$ kG, one can use CO_2-laser at 10.6 μ. Third, the nonlinear cyclotron resonance in semiconductors is accompanied by the appearance of a voltage drop between the two surfaces of the sample facing the radiation. The voltage drop arises from the redistribution of rotating carriers under the influence of the radiation pressure and exhibits bistability. Therefore, the proposed effect could be the first known all-optical nonlinear phenomenon which yields an opto-electronic bistability.

Consider a semiconductor with a conduction band [‡1] which has a relativistic momentum dispersion as in Kane's isotropic two-band model [‡2] [2],

[‡1] The argument is easily generalized to holes of a valence band with a relativistic momentum dispersion.

[‡2] This type of semiconductors have been discussed extensively for the purpose of obtaining a cyclotron maser. See refs. [3, 4].

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$$E_c(p) = [(m_0^*)^2 v_0^4 + p^2 v_0^2]^{1/2} ,$$

where $\boldsymbol{p}$ is momentum, m_0^* is the effective mass at the band edge, $v_0 = (E_G/2m_0^*)^{1/2}$ is a charactistic speed, and E_G is the band gap. The velocity of a conduction electron is given by $\boldsymbol{v} = \partial E_c/\partial \boldsymbol{p}$, and therefore, $\boldsymbol{p} = m_0^* \gamma \boldsymbol{v}$, where $\gamma = (1 - v^2/v_0^2)^{-1/2}$. For narrow gap semiconductors like $Pb_{1-x}Sn_xTe$, $Pb_{1-x}Sn_xTe$, or $Cd_{1-x}Hg_xTe$, which have such relativistic bands, typically [5] $E_G \sim 0.05$ eV, $m_0^* \sim 10^{-2} m$ (m is the bare electron mass), and $v_0 \sim 6 \times 10^7$ cm/s. For InSb, $E_G \sim 0.24$ eV, $m_0^* \sim 0.014\, m$, and $v_0 \sim 10^8$ cm/s.

The semiconductor is placed in a homogeneous field $\boldsymbol{B}_0$ and irradiated by a circularly polarized light $\boldsymbol{E}_r$ propagating parallel to $\boldsymbol{B}_0$. To simulate fields associated with redistribution of free carriers inside the semiconductor, we assume an externally applied voltage which causes an electrostatic field of the form $\boldsymbol{E}_s = -g\hat{\boldsymbol{z}}$. The total electric and magnetic fields are

$$\boldsymbol{E} = \boldsymbol{E}_r + \boldsymbol{E}_s = E[\hat{\boldsymbol{x}} \sin(\omega t - kz) + \hat{\boldsymbol{y}} \cos(\omega t - kz)] - g\hat{\boldsymbol{z}} , \quad (1a)$$

$$\boldsymbol{B} = \boldsymbol{B}_r + \boldsymbol{B}_0 = E[-\hat{\boldsymbol{x}} \cos(\omega t - kz) + \hat{\boldsymbol{y}} \sin(\omega t - kz)] + B_0\hat{\boldsymbol{z}} , \quad (1b)$$

where $k = \omega\sqrt{\epsilon}/c$, and ϵ is the dielectric constant of the material at frequency ω. The classical equation of motion for an electron is

$$\mathrm{d}\boldsymbol{p}/\mathrm{d}t = e\boldsymbol{E} + (e/c)\boldsymbol{v} \times \boldsymbol{B} - \Gamma \boldsymbol{p} . \quad (2)$$

Here $\Gamma \boldsymbol{p}$ is a damping term and Γ usually depends on p. To simplify the following, however, we assume that Γ is constant. In (2), the term

$$(e/c)\boldsymbol{v} \times \boldsymbol{B}_r \sim (e/\omega)[\boldsymbol{v} \times (\boldsymbol{k} \times \boldsymbol{E}_r)]$$

represents the radiation pressure on the rotating electron. It is usually neglected [4,6–8]. Ref. [9] is an exception, although losses and other forces which might arise from population redistribution were not considered there either.

In a completely self-consistent treatment, eq. (2) should be supplemented with another equation for g which comes from a self-consistent, multiparticle treatment of the dynamics of population redistribution. This will not be undertaken here. However, one can get a fair idea of the time scale on which such a redistribution takes place from the analogy to the Hall effect. As in the Hall effect [10], one can expect that positive and negative charges are localized on the two opposing surfaces which face the radiation, while the charge distributions inside the material remain uniform and neutral. Thus, the sample can be considered as a capacitor with capacitance $C = \epsilon_0/4\pi L$, where ϵ_0 is the static dielectric constant and L is the sample thickness. g is related to the capacitor charge density by $g = Q/CL = 4\pi Q/\epsilon_0$. Q is coupled to the z-component of the total electric field by $\mathrm{d}Q/\mathrm{d}t = \sigma E_z$, where σ is the conductivity. It follows that $\tau_0 = \epsilon_0/4\pi\sigma$ is the relaxation time for charge redistribution. In the following we consider the state of the system for $t \gg \tau_0$.

One can obtain a steady state solution to (2) by letting

$$p_z = 0 ,$$

$$\boldsymbol{p}_\perp = p[\hat{\boldsymbol{x}} \sin(\omega t - kz + \phi) + \hat{\boldsymbol{y}} \cos(\omega t - kz + \phi)] . \quad (3)$$

The steady state equations are then

$$I/I_0 = (p/p_0)^2 [1 + (\omega/\Gamma - \omega_c^*/\gamma\Gamma)^2] , \quad (4a)$$

$$g/g_0 = \gamma^{-1}(p/p_0)^2 , \quad (4b)$$

and

$$\cot\phi = \omega/\Gamma - \omega_c^*/\gamma\Gamma , \quad (4c)$$

where I is the radiation intensity, $p_0 = m_0^* v_0$ is a characteristic momentum of the band, $\gamma = [1 + (p/p_0)^2]^{1/2}$ is the usual relativistic factor written in terms of momentum, $I_0 = c m_0^* E_G \Gamma^2/8\pi e^2$ is a characteristic intensity, $\omega_c^* = eB_0/m_0^* c$ is the cyclotron frequency at the band edge, and $g_0 = E_G\Gamma/2ec$ is a characteristic voltage drop per unit length. Figs. 1 and 2 show the plots of $(p/p_0)^2$ and (g/g_0) versus (I/I_0) for various ω. In fig. 1, the sections of the curves with $\mathrm{d}p^2/\mathrm{d}I > 0$ are stable, and those with $dp^2/\mathrm{d}I < 0$ are unstable. It is seen that for a given ω, the semiconductor exhibits bistability if

$$I_0[(\omega_c^*/\omega)^2 - 1] \approx I_1 < I < I_2 \approx I_0(\omega/\Gamma)^2[(\omega_c^*/\omega)^{2/3} - 1]^3 . \quad (5)$$

The estimates for I_1 and I_2 are obtained from the singularity of $\mathrm{d}p^2/\mathrm{d}I$ at a hysteretic jump. The same analysis also shows that in order to have hysteretic behavior, ω and ω_c^* must be such that

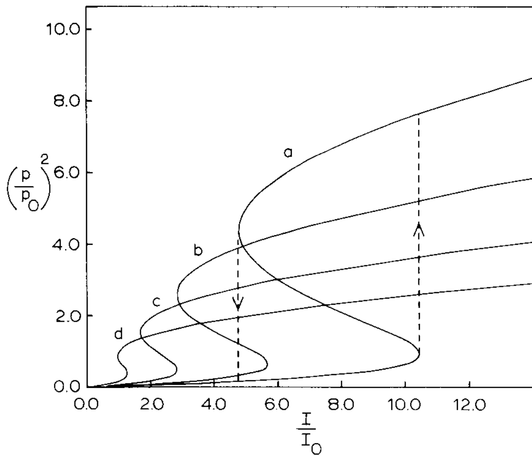

Fig. 1. $(p/p_0)^2$ versus I/I_0 for $\omega_c^* = 10\Gamma$: (a) $\omega = 4\Gamma$; (b) $\omega = 5\Gamma$; (c) $\omega = 6\Gamma$; (d) $\omega = 7\Gamma$.

$$[(\omega_c^*/\omega)^{4/3} + (\omega_c^*/\omega)^{2/3} + 1]\,[(\omega_c^*/\omega)^{2/3} - 1]^{-2} \lesssim (\omega/\Gamma)^2 < (\omega_c^*/\Gamma)^2 . \qquad (6)$$

Of course, to observe the cyclotron resonance, one must have $\Gamma \ll \omega_c^*$, ω.

It is seen from fig. 1 that, as I is increased from zero, the system moves along the lower branch of a curve corresponding to ω until I reaches I_2. At $I = I_2$, the system makes a hysteretic jump to the stable upper branch and moves along the upper branch with increasing I. Prior to the jump at I_2, the momentum is approximately given by

$$p_2 \approx p_0[(\omega_c^*/\omega)^{2/3} - 1]^{1/2} .$$

If I is decreased now, the system moves down the upper branch past I_2 to I_1. At $I = I_1$, the system jumps to the lower stable branch. Prior to the jump, the momentum is

$$p_1 \approx p_0[(\omega_c^*/\omega)^2 - 1]^{1/2} .$$

After the jump, the momentum is

$$p_3 \approx (p_0\Gamma/\omega)(\omega_c^* + \omega)^{1/2}(\omega_c^* - \omega)^{-1/2} .$$

Note that the hysteretic jumps at I_1 and I_2 are contingent upon the charge distribution being in steady state, at least relative to cyclotron transitions. In other words, the charge relaxation must be fast such that $\tau_0\Gamma \ll 1$. Furthermore, variations in I must be slow such that $\tau_0 \mathrm{d}I/\mathrm{d}t \ll I$. Using the expression for mobility $\mu = e/m^*\Gamma$ and the conductivity $\sigma = eN\mu$, where N is the free carrier density, one can write

$$\tau_0\Gamma = \epsilon_0 m^*\Gamma^2(4\pi e^2 N)^{-1} = (\epsilon_0/\epsilon_\infty)(\Gamma/\omega_p)^2 \ll 1.$$

ω_p is the plasma frequency. This condition puts a lower limit on the carrier density such that [‡3]

$$N \gg N_1 = \epsilon_0 m^*\Gamma^2(4\pi e^2)^{-1} .$$

If $N \lesssim N_1$, which corresponds to $\tau_0\Gamma \gtrsim 1$, then charge oscillations may be excited and the system may not reach a stable steady state.

There is also an upper limit on the free carrier density N. This comes about from the fact that at low temperatures, band states are filled up to a Fermi momentum p_F and cyclotron transitions should excite the electron to states for which $p > p_F$. One can see from the curves in fig. 1 that the transition from the higher branch to the lower cannot occur if $p_3 < p_F$. Therefore, in order to observe the bistable behavior in full, one must have $p_3 > p_F$, and since $p_F = \hbar(3\pi^2 N)^{1/3}$ as temperature goes to zero,

$$\epsilon_0 m^*\Gamma(4\pi e^2)^{-1} = N_1 \ll N < N_2 = (3\pi^2\hbar^3)^{-1}p_0^3[(\omega_c^*/\omega)^{2/3} - 1]^{3/2} . \qquad (7)$$

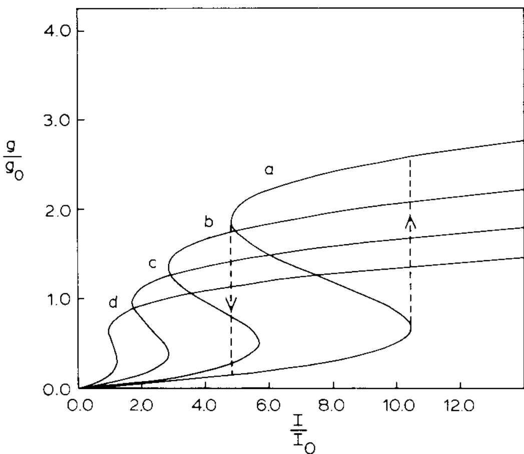

Fig. 2. g/g_0 versus I/I_0 for $\omega_c^* = 10\Gamma$. a, b, c and d are the same as in fig. 1.

‡3 m^* here is the appropriate effective mass near the Fermi surface. Typically $m^* \sim 4m_0^*$.

Eqs. (5)–(7) define a critical region in which the bistability can be observed. Finally, fig. 2 shows the bistability of the voltage drop gL along the sample.

To get a feeling for the numbers, consider InSb. From the mobility measurements one has $\Gamma \approx 2 \times 10^{10}$ s^{-1} for temperature in the range $10\ K < T < 100\ K$ [12] (at room temperatures $\Gamma \approx 8 \times 10^{10}\ s^{-1}$ [13]). Using the low temperature value, $I_0 \sim 1.2\ W/cm^2$, $g_0 \sim 0.8$ V/cm and $N_1 \approx 10^{12}\ cm^{-3}$. Assuming $1.2 < \omega_c^*/\omega < 2$, one has $4 \times 10^{15}\ cm^{-3} \leqslant N_2 \leqslant 4 \times 10^{16}$ cm^{-3}. Apparently one needs relatively pure crystals. Note that the upper limit on the free carrier density presents the most difficulty from the experimental point of view, since even in relatively pure crystals hot carriers may be created under steady illumination, which may cause the free carrier density to increase due to pair production [14]. The numbers above are nevertheless quite favorable for an experiment.

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