Extreme-ultraviolet and x-ray emission and amplification by nonrelativistic electron beams traversing a superlattice

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High-energy electrons emit resonant electromagnetic radiation when passing through a spatially periodic medium. It is conventionally assumed that ultrarelativistic electron beams are required to obtain significant emission. We demonstrate theoretically the feasibility of exploiting solid-state superlattices with short spatial periods to obtain both spontaneous and stimulated emission in the extreme-ultraviolet and soft x-ray range using nonrelativistic beams.

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Fast-moving electrons emit electromagnetic waves when moving from one medium into another with a different dielectric constant.[1-3] This is known as transition radiation and was predicted by Ginzburg and Frank.[1] In a spatially periodic medium the waves emitted at different interfaces interfere so that a resonant emission is obtained when the following condition is satisfied[2-4]:

$$\sqrt{\bar{\epsilon}}\cos\theta = c/v - n\lambda/l, \qquad (1)$$

where λ is the wavelength of radiation, l the period of the spatially varying dielectric constant ϵ of medium (it is conventionally assumed that the variations are very small), v velocity of the electrons (assumed normal to the interfaces), θ the angle between the direction of wave propagation and electron motion, n an integer, and $\bar{\epsilon}$ a "mean" ϵ. Usually the period $l \gg \lambda$, so that ultrarelativistic beams ($v/c \simeq 1$) are required in order to satisfy Eq. (1) for real θ. Recently, the possibility of stimulated resonance radiation of ultrarelativistic (~ 50 GeV) electrons traveling through a stack of metal foils was considered,[4] with $l \sim 7$ cm.

In this letter we demonstrate the feasibility of using nonrelativistic electron beams in order to attain both spontaneous and stimulated emission in the extreme ultraviolet and x-ray range using solid-state superlattices with $l \sim 100$ Å so that $l/n\lambda \sim 1$. We show that the wavelength of resonant radiation and the required energy of electrons are determined by the parameter $Q = n\lambda_p/l$, where λ_p is a "mean" plasma wavelength of the medium. If $Q \ll 1$, as is actually assumed in all previous work,[2-4] then the wavelength of the resonant radiation has an order of magnitude of $\lambda \sim \lambda_p Q$, and the kinetic energy of the electrons eU/mc^2 must exceed the critical amount $\sim 1/\sqrt{Q^2 - \theta^2} \gg 1 (\theta < Q)$ which constitutes the use of ultrarelativistic beams. On the contrary, if the period of the spatially periodic medium is chosen so that $Q \gg 1$, the wavelength of resonant radiation becomes of the order $\sim \lambda_p/Q = l/n$, and the critical kinetic energy of the beam turns out to be extremely small: $eU/mc^2 \sim 1/2Q^2$. The advantages of the proposed method are (1) the frequency of radiation can be easily tuned in a very wide range by simply varying accelerating potential of beam (which is very hard to do with ultrarelativistic beams), (2) the range of the angles of the radiated emission is almost unlimited, and (3) the cost of equipment and energy required for experiments with nonrelativistic beams is insignificant compared to large accelerating machines. This last consideration is perhaps the most important.

Fortunately, the development of molecular beam epitaxy (MBE) and other techniques in recent years has made it possible to grow very thin films (~ 100 Å and less) with precise boundaries. Periodic structures composed of thin films of different materials (in particular, superlattices) have also been fabricated.[5] Using these structures, nonrelativistic electron beams with energies 70–200 keV can be used to generate radiation of wavelength 100–200 Å and less. An important feature of transition radiation is that the electron beam travels *through* the material structure rather than in vacuum *above* the structure (as in traveling wave tubes[6]), or through a spatially modulated magnetic field (as in many free-electron lasers[7]). This makes it possible to use very short spatial periods.

We will first obtain the resonant wavelength of radiation from Eq. (1) noting that[3] $l\sqrt{\bar{\epsilon}} = l_1\sqrt{\epsilon_1} + l_2\sqrt{\epsilon_2}$, where $l_{1,2}$ and $\epsilon_{1,2}$ are, respectively, the thicknesses and dielectric constants of alternating layers forming the superlattice; $l = l_1 + l_2$ is its spatial period. For short wavelengths ($\lambda^2 \ll \lambda_{1,2}^2$), $\epsilon_{1,2} = 1 - \lambda^2/\lambda_{1,2}^2$, where $\lambda_{1,2}$ are the plasma wavelengths of the two materials forming the superlattice ($\lambda_{1,2}^2 = 4\pi mc^2/e^2 N_{1,2}^e$, where N^e is the density of electrons). Thus, the mean dielectric constant may be written as $\bar{\epsilon} \simeq 1 - \lambda^2/\lambda_p^2$, where $\lambda_p^{-2} = l^{-1}(l_1\lambda_1^{-2} + l_2\lambda_2^{-2})$. Substituting this into Eq. (1) and solving it for λ, one gets the resonant wavelengths:

$$\lambda = \lambda_p \frac{Q}{Q^2 + \cos^2\theta}\left[\frac{1}{\beta} \pm \frac{\cos\theta}{Q} \times \sqrt{(\gamma_{cr}^2 - 1)^{-1} - (\gamma^2 - 1)^{-1}}\,\right], \qquad (2)$$

where $\beta = v/c$, $\gamma = (1 - \beta^2)^{-1}$, and $\gamma_{cr} = [1 + (Q^2 - \sin^2\theta)^{-1}]^{1/2}$ is the critical energy required for the excitation of resonant radiation. For $Q \ll 1$, $\lambda \simeq \lambda_p (Q \pm \sqrt{\gamma_{cr}^{-2} - \gamma^{-2}})$. Here, $\gamma_{cr} \gg 1$, such that only an ultrarelativistic beam can excite radiation. On the other hand, when $Q \gg 1$, the critical kinetic energy turns out to be extremely low, $\gamma_{cr} - 1 \simeq 1/2Q^2$, which gives less than 10 keV for all conventional materials if $l \sim 100$ Å. For sufficiently higher (but still nonrelativistic) energies eU, Eq. (2) gives simply

$$\lambda \simeq \frac{l}{n}\left(\frac{1}{\beta} - \cos\theta\right) \simeq \frac{l}{n}\left(\sqrt{mc^2/2eU} - \cos\theta\right);$$
$$eU \ll mc^2 = 0.51 \text{ MeV}. \qquad (3)$$

For instance, if $l = 100$ Å, $n = 1$, $eU = 75$ keV, and $\theta = 45°$, one has $\lambda = 113.6$ Å, for $n = 10$ one has $\lambda = 11.3$ Å.

The resonant radiation (i.e., spontaneous emission in the system) can provide a narrowband source of radiation. A single electron traversing multilayer structure with N layers radiates energy I in a solid angle $d\Omega$ in the frequency interval between ω and $\omega + d\omega$, given by[1-4]

$$\frac{d^2I}{d\omega\, d\Omega} = \left(\frac{d^2I_0}{d\omega\, d\Omega}\right)\frac{4\sin^2(\pi n l_1/l)\sin^2(\xi N/2)}{\sin^2\xi}, \qquad (4)$$

where $\xi = (1/\beta - \sqrt{\bar\epsilon}\cos\theta)\pi l/\lambda$, and $d^2I_0/d\Omega\, d\omega$ is a radiation produced by a single interface. According to the Ginsburg–Frank theory,[1-3] for nonrelativistic electrons ($\beta^2 \ll 1$) and small variations of $\epsilon(|\Delta\epsilon| = |\epsilon_1 - \epsilon_2| \ll \bar\epsilon)$ the distribution of single-interface radiation is given by

$$d^2I_0/d\Omega\, d\omega = e^2\beta^2[\Delta\epsilon(\omega)]^2\sin^2\theta/4\pi^2c. \qquad (5)$$

If the number N of layers is sufficiently large, ($N \gg |\bar\epsilon/\Delta\epsilon|$), Eq. (4) provides for very narrow spectral peaks of radiation for each particular angle θ [with central wavelength determined by Eq. (3)], which also implies that any frequency is radiated in very narrow intervals of angles. Noting that $\Delta\epsilon = \lambda^2(\lambda_1^{-2} - \lambda_2^{-2})$, and integrating Eq. (4) over ω and Ω (with $d\Omega = 2\pi\sin\theta\, d\theta$), one gets the total radiation in each order n

$$I \simeq \frac{16e^2Ll^2(\lambda_1^{-2} - \lambda_2^{-2})^2\sin^2(n\pi l_1/l)}{3\beta n^4\pi}, \qquad (6)$$

(where $L = Nl/2$ is the total thickness of the structure) with the wavelengths of radiation being in the range

$$\frac{l}{n}\left(\frac{1}{\beta} - 1\right) < \lambda < \frac{l}{n}\left(\frac{1}{\beta} + 1\right).$$

The total energy of radiation increases as speed β decreases. In order to calculate the power of resonant radiation emitted by an electron beam with an electrical current J one must multiply Eqs. (4)–(6) by J/e. If $l = 100$ Å, $l_1 = l_2 = l/2$, $L = 1\,\mu$m, $eU = 75$ keV, $J = 1$ mA, and $\lambda_1 \simeq 400$ Å (e.g., Zn, Cu, Ag, or Au), $\lambda_2 \simeq 800$ Å (e.g., Si or Ge, see Ref. 8), the system can provide a radiation of first harmonic ($n = 1$) with a total power ~ 0.1 mW and a mean wavelength ~ 200 Å.

We will derive now an amplification caused by the stimulated emission. This effect may be viewed in the following way. An em wave having a wave vector component $k_z = k_0\cos\theta$ along the axis z (which coincides with the electron trajectory) produces the higher order spatial harmonics with $k_{z_n} = k_z \pm 2\pi n/l$ which is due to the periodicity of medium. The phase velocities of these harmonics along the axis x are, therefore, $v_n = c/\sqrt{\bar\epsilon}(\cos\theta \pm 2\pi n/lk_0)$. If the resonant condition [Eq. (1)] for λ is fulfilled, one of these phase velocities coincides with the speed of the electron that results in an exchange of energy between the em wave and the electron. For some frequencies in the neighborhood of resonance, the electron loses energy to the em wave; this results in a coherent gain of the wave, or stimulated emission. The important point is to find the intensities of the resonant spatial harmonics of the field. In all the previous work on resonant transition radiation[2-4] it is assumed that $\lambda \ll l$ (which is always valid in the ultrarelativistic case; see the introductory section). This allows one to use the WKB approximation. This approximation is not valid on our case since λ may be of the order or longer than l. Instead, we will find a solution of the exact wave equation (with periodic parameters) based on the assumption of smallness of variations of susceptibility (i.e., $|\Delta\epsilon/\bar\epsilon| \ll 1$, which is always true for short wavelengths); no assumption is made regarding the ratio λ/l. In this letter, we approach the problem using a single-particle picture which provides direct insight into the mechanism of the electron-em wave interaction. The problem can also be treated using either the Boltzmann equation[4] or a quantum mechanical formalism; we plan to address these aspects in a subsequent publication.[9]

We consider the exact Maxwell equation for the em field and $\epsilon(z)$ being an arbitrary periodic function in z. We assume a plane wave; it can be shown that only the em wave with its electric field $\bar{\mathbf{E}}$ polarized in the plane of incidence (i.e., plane x,z) may be amplified by the beam.[10] By virtue of the Floquet's theorem for wave equations with periodic coefficients,[11] any component of the em field can be written as a sum of spatial harmonics:

$$u = u_0\exp(j\mathbf{k}\cdot\mathbf{r} - j\omega t)\left[1 + \sum_{n\neq 0}\rho_n\exp\left(\frac{2jn\pi z}{l} + j\phi_n\right)\right], \qquad (7)$$

where $\mathbf{k}\cdot\mathbf{r} = k_0x\sin\theta + k_0z\cos\theta$ and ρ_n is the amplitude of the nth spatial harmonic. We make the conventional assumption that there is no retroreflection, which is valid if $N|\Delta\epsilon/\bar\epsilon|^2 \ll 1$ and $|\Delta\epsilon/\epsilon_0| \ll \cos\theta$. This assumption is strictly true in the vicinity of $\theta = 45°$ (see Ref. 10). Substituting the em field in the form of Eq. (7) into the Maxwell equations, collecting the terms with $\exp(j\mathbf{k}\cdot\mathbf{r} + 2jn\pi z/l)$ for each particular n and retaining only terms that are first order in a_n ($\ll\cos\theta$), where the a_n's are the Fourie coefficients of $\epsilon(z)$: $\epsilon(z) = \bar\epsilon + \Sigma_{n=1}^{\infty} a_n\cos(2n\pi z/l + \psi_n)$, one gets the amplitudes u_0, ρ_n of the spatial harmonics of nonvanishing components of electric and magnetic fields. The z component of the electric field E_z is most important for the stimulated emission; for this component one gets

$$E_{z_0} = -E_0\sin\theta;$$
$$\rho_{z_n} = a_n(1 - q\cos\theta - q^2)/2q(2\cos\theta + q);$$
$$q = \lambda n/l\sqrt{\bar\epsilon}. \qquad (8)$$

where E_0 is the amplitude of the principal harmonic of total electric field. Further calculations are based on the conventional model of energy exchange between the em field and an electron which is used, e.g., in the theory of free-electron lasers (see, e.g., Ref. 12). From the Lorentz equation $mc\,d(\boldsymbol\beta\gamma)/dt = e(\mathbf{E} + [\boldsymbol\beta\times\mathbf{H}])$, one gets the equation for the energy $\mathscr{E} = \gamma mc^2$ of electron

$$d\mathscr{E}/dt = ec(\boldsymbol\beta\mathbf{E}) = ec(\tilde{\boldsymbol\beta}\tilde{\mathbf{E}} + \tilde{\mathbf{E}}\Delta\boldsymbol\beta + \tilde{\boldsymbol\beta}\Delta\mathbf{E}), \qquad (9)$$

where $\mathbf{E} = \mathbf{E}[\mathbf{r}(t), t]$ is the field at the instantaneous location of the electron, $\tilde{\boldsymbol\beta}$ and $\tilde{\mathbf{E}}$ unperturbed vectors, $\Delta\boldsymbol\beta$ a small perturbation of electron velocity due to interaction, and $\Delta\mathbf{E}$ a small perturbation of the field seen by the electron due to its spatial displacement Δz in respect to the unperturbed trajectory, i.e.,

$$\Delta \mathbf{E} = \frac{\partial \widetilde{\mathbf{E}}}{\partial z} \Delta z(t); \Delta z = c \int_0^t \Delta\beta_z dt; \Delta\beta_z = \frac{e}{\gamma^2 mc} \int_0^t E_z \, dt. \tag{10}$$

In Eqs. (9) and (10) one has to take into account only that particular nth component of the wave which is "resonant" to the speed of electron, i.e., that one with $|\xi - n\pi| \ll 1$. After substituting $z = c\beta t$ and the amplitudes Eq. (8) of the proper resonant harmonic of E_z, into Eqs. (9) and (10), and integrating over the temporal interval $[0, \tau = L/\beta c]$, where τ is a time for an electron to pass through the superlattice, one has to average the result over all the possible phases ϕ_n of the relevant field harmonic. We denote this operation by angle brackets. Note that the term $\langle \widetilde{\beta \mathbf{E}} \rangle$ in Eq. (9) vanishes. Finally, one gets the total averaged change of the electron energy per pass for the nonrelativistic case:

$$\langle \Delta \mathscr{E} \rangle_n = \pi e^2 E_0^2 L^3 \rho_{z_n}^2 \times \frac{\sin^2\theta(-2 + 2\cos\nu_n\tau + \nu_n\tau\sin\nu_n\tau)}{m\beta^3 c^2 (\nu_n \tau)^3 \lambda}, \tag{11}$$

where $\nu_n = \beta c(n\pi - \xi)/l$ is a resonant factor and $\rho_{z_n}^2 \simeq (a_n/2)^2(1 - \beta\cos\theta)^2$. For some ν_n, the change of energy $\langle \Delta \mathscr{E} \rangle$ becomes negative which constitutes the gain of em field, $\langle \Delta \mathscr{E}_{\rm em} \rangle = -\langle \Delta \mathscr{E} \rangle$. Replacing the term $(-2 + 2\cos\nu\tau + \nu\tau\sin\nu\tau)/\nu^3\tau^3$ by its negative extremum $-4/\pi^3$ $(\nu\tau \simeq \pi)$, one gets the maximal em-wave gain per electron per pass:

$$\langle \Delta \mathscr{E}_{\rm em} \rangle = a_n^2 e^2 E_0^2 L^3 \sin^2\theta (1/\beta - \cos\theta)^2 / \pi^2 mc^2 \beta\lambda. \tag{12}$$

In order to obtain an amplification Γ per pass in the system bombarded by an electron beam with the density of electric current i(A/cm^2), one has to multiply $\langle \Delta \mathscr{E}_{\rm em} \rangle$ by i/e and divide by the energy flux of incident em wave per unity area of the interface $E_0^2 \cos\theta/2R$, where $R = 377\ \Omega$ is the vacuum impedance. One has also to take into account that for rectangular form of $\epsilon(z)$, $a_n/2 = (\Delta\epsilon/n\pi)\sin(n\pi l_1/l)$ with $\Delta\epsilon = \lambda^2(\lambda_1^{-2} - \lambda_2^{-2})$. The maximal em wave amplification per pass is

$$\Gamma = \frac{8\mu ieRL^3 \sin^2(\pi n l_1/l)\sin^2\theta}{mc^2 l \pi^4 \cos\theta}, \tag{13}$$

where $\mu = \beta^{-1} l^4 (\lambda_1^{-2} - \lambda_2^{-2})^2 (1/\beta - \cos\theta)^5 n^{-5} = \lambda^5 l^{-1} (\lambda_1^{-2} - \lambda_2^{-2})^2 (\cos\theta + n\lambda/l)$. If $\lambda = 140$ Å, $l = 100$ Å, $l_1 = l_2 = 50$ Å, $n = 1$, $L = 1$ μm, $\lambda_1 \simeq 400$ Å, $\lambda_2 \simeq 800$ Å, $\theta = 45°$, and $i = 5 \times 10^{10}$ A/cm^2 (Ref. 4) (e.g., beam of 2 μm diameter with a current $\sim 1.5 \times 10^3$ A), one gets an amplification $\Gamma \simeq 5\%$ per pass. The required speed of electrons is $\beta \simeq 0.474$ which corresponds to energy $eU = 69$ keV. With the mirrors situated outside the superlattice to form a Fabry–Perot resonator to provide feedback, the system becomes a short-wave laser. It is obvious that the amplifiers and lasers based on the proposed principle should work in the very short pulse regime of operation, with the duration of current pulse being determined by the heating, ionization, diffusion of absorbed electrons, etc. As a rough approximation, the per atom heating rate caused by the energy losses of the electron beam,[13] is

$$\frac{(n^e/N^a) d\mathscr{E}}{dt} = 4\pi i Z (mv^2)^{-1} \ln\left(\frac{\gamma^2 mv^3}{e^2\omega}\right), \tag{14}$$

where the $n^e = i/ev$ is the electron beam density, N^a the atomic density of the material, and Z the atomic number. For the parameters mentioned above, the duration of the current pulse must be shorter than $\sim 10^{-13}$ s in order for the energy transfer per atom to be of order ~ 1 eV or less. One may note though that in the case of ultrarelativistic beams[4] with energy ~ 50 GeV for the same current, the losses are even greater, such that one needs even shorter pulses. It will probably be hard to obtain such short and powerful pulses, but it may prove worthwhile. It should be noted that coherent sources are not available at these short wavelengths and getting significant stimulated emission is a difficult problem in general. However, the first step is to attain spontaneous resonant emission as described in the beginning of this letter. The spontaneous emission can be used as a very narrowband source of radiation by selecting narrow range of angles [Eq. (4)]. It should also be noted that the spontaneous radiation intensity depends on the total current in the electron beam unlike the stimulated emission gain which depends on the current density. Therefore, the conditions to obtain spontaneous emission are much more relaxed. According to Eqs. (4)–(6), the electrical current required to observe spontaneous emission with the same spot size of the beam is 10^6–10^9 less than that required for stimulated emission.

In conclusion, we have demonstrated the feasibility of generating extreme-ultraviolet and soft x-ray radiation by electron beams with relatively low, nonrelativistic energies, traversing the solid-state superlattice composed of very thin periodic layers. The proposed system can be used as a very efficient noncoherent source of narrowband radiation, and, under special conditions, as an amplifier and laser.

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[1]V. L. Ginzburg and I. M. Frank, Zh. Eksp. Teor. Fiz. **16**, 15 (1946) (in Russian).
[2]G. M. Garibyan, Sov. Phys. JETP **33**, 23 (1971).
[3]M. L. Ter-Mikaelian, *High Energy Electromagnetic Processes in Condensed Media* (Wiley–Interscience, New York, 1972).
[4]M. A. Piestrup and P. F. Finman, IEEE J. Quantum Electron. **QE-19**, 357 (1983).
[5]See, for example, A. C. Gossard, Thin Solid Films **57**, 3 (1979).
[6]See, e.g., A. H. W. Beck, *Space-Charge Waves* (Pergamon, New York, 1958), Chap. 7.
[7]See, e.g., A. Gover and P. Sprangle, IEEE J. Quantum Electron. **QE-17**, 1196 (1981).
[8]P. M. Platzman and P. A. Wolff, *Waves and Interactions in Solid State Plasmas* (Academic, New York, 1973).
[9]S. Datta and A. E. Kaplan (unpublished).
[10]Incidentally, this fact brings up a substantial advantage. For the polarization in the plane of incidence, there is an angle θ at which the retroreflection at the layer interfaces vanishes altogether; it is the so-called Brewster angle. In our case $(|\Delta\epsilon| \ll \bar{\epsilon})$ this angle is $\theta \sim 45°$.
[11]S. M. Rytov, Zh. Eksp. Teor. Fiz. **29**, 605 (1955) (in Russian) [Sov. Phys. JETP **2**, 466 (1956)].
[12]A. Fruchtman, J. Appl. Phys. **54**, 4289 (1983).
[13]J. D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1962), Eq. (13.13).