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Soft X-ray Emission Excited by Electron Beams in a Superlattice

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High-energy electrons emit resonant electromagnetic radiation when passing through a spatially periodic medium. It is conventionally assumed that ultra-relativistic electron beams are required to obtain significant emission. We demonstrate theoretically the feasibility of exploiting solid-state superlattices with short spatial periods to obtain both spontaneous and stimulated emission in the extreme-ultraviolet and soft X-ray range using *non-relativistic* beams. Both classical and quantum approaches are developed.

1. Introduction

Fast moving electrons emit EM waves when passing through a spatially periodic medium (resonant transition radiation).[1,2] The wavelength λ of the radiation emitted at an angle θ with respect to the electron trajectory is determined by the formula $\bar{n}(\lambda)\cos\theta = \beta^{-1} - r\lambda/\ell$, where $\beta = v/c$ is dimensionless speed at the electron, $\bar{n}(\lambda)$ is averaged refractive index of the system, r is an integer, and ℓ is the period of spatial modulation. It is conventionally assumed that ultra-relativistic beams (e.g., up to 50 GeV, see Ref [3]) are required to attain this kind of emission. We show[4] that if the period ℓ is much shorter than a "mean" plasma wavelength of the medium λ_p (which can be done by using solid-state superlattices with the spatial period 50–200Å), the critical kinetic energy required to get a radiation, turns out to be extremely low. Indeed, for wavelengths λ much shorter than the plasma wavelength λ_p (averaged over layers), the averaged refractive index is $\bar{n} \simeq 1-\lambda^2/2\lambda_p^2$, and the resonant wavelength of radiation is:[4]

$$\lambda = \lambda_p \frac{Q}{Q^2+\cos^2\theta}\left(\frac{1}{\beta} \pm \frac{\cos\theta}{Q}\sqrt{(\gamma_{cr}^2-1)^{-1}-(\gamma^2-1)^{-1}}\right), \quad (1)$$

where $Q = r\lambda_p/\ell$; $\gamma = (1-\beta^2)^{-1}$, and $\gamma_{cr} = [1+(Q^2-\sin^2\theta)^{-1}]^{1/2}$ is the critical energy required for the excitation of resonant radiation. For $Q << 1$, $\gamma_{cr} >> 1$, such that only an ultra-relativistic beam can excite radiation. On the other hand, when $Q >> 1$, the critical kinetic energy is very low, $\gamma_{cr}-1 \simeq 1/2Q^2$, which corresponds to less than 10 KeV for all conventional materials if $\ell \sim 100$ Å. Multilayer structures with such a short period (and even shorter) can be realized either by superlattices (e.g., formed by molecular beam epitaxy[5]) or by multilayer coatings for X-ray mirrors[6] (with $\ell \simeq 20$–30Å). Using these structures, one can get a significant radiation in the range 10Å–300Å using non-relativistic beams with energies 70-300 KeV. The main advantage of the proposed method is that the cost of equipment and energy required for experiments with non-relativistic beams is insignificant compared to large accelerating machines.

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2. Spontaneous Emission

The spontaneous radiation from the system has a conical structure with the emission wavelength changing with angle. A single nonrelativistic electron traversing multilayer structure with N layers radiates energy I in a solid angle $d\Omega$ in the frequency interval between ω and $\omega + d\omega$,[1,2]

$$d^2I_o/d\Omega d\omega = e^2\beta^2[\Delta\epsilon(\omega)]^2\sin^2\theta\sin^2(\pi r\ell_1/\ell)\sin^2(\xi N/2)/\sin^2\xi\cdot\pi^2c \qquad (2)$$

where $\xi = (\beta^{-1} - \bar{n}\cos\theta)\pi\ell/\lambda$, ℓ_1 is the thickness of the first layer, and $\Delta\epsilon = \lambda^2(\lambda_1^{-2} - \lambda_2^{-2})$, λ_1 and λ_2 are the plasma frequencies of the respective layers. If the number N of layers is sufficiently large, ($N >> |\bar{n}/\Delta\epsilon|$), Eq(2) provides for very narrow spectral peaks of radiation for each particular angle θ [with central wavelength $\lambda \simeq \ell r^{-1}(\beta^{-1}-\cos\theta)$]. The total radiation power I in each order r per electron is

$$I \simeq 16e^2L\ell^2(\lambda_1^{-2}-\lambda_2^{-2})^2\sin^2(\pi r\ell_1/\ell)/3\beta r^4\pi \qquad (3)$$

the wavelengths of radiation being in the range $\ell r^{-1}(\beta^{-1}-1) < \lambda < \ell r^{-1}(\beta^{-1}+1)$. ($L = N\ell/2$ is the total thickness of the structure). If $\ell = 100Å$, $\ell_1 = \ell_2 = \ell/2$, $L = 1\mu m$, $eU = 75KeV$, $J = 1mA$ where J is the total current, and $\lambda_1 \simeq 400Å$, $\lambda_2 \simeq 800Å$, the system can provide a radiation of first harmonic ($r = 1$) with a total power ~ 0.1 mW and a mean wavelength ~ 200 Å.

3. Stimulated Emission

If the total current density i of the electron beam is sufficiently large, the system can provide stimulated emission and amplification. The effect of amplification in the multilayer system may be viewed in the following way. An EM wave having a wave vector component $k_z = k_o\cos\theta$ along the electron trajectory, produces the higher order spatial harmonics with $k_{z_r} = k_z \pm 2\pi r/\ell$ which is due to the periodicity of medium. The phase velocities of these harmonics along that trajectory are, therefore, $v_r = c/\bar{n}(\cos\theta \pm 2\pi r/\ell k_o)$. If the resonant condition for λ is fulfilled, one of these phase velocities coincides with the speed of the electron that results in an exchange of energy between the EM wave and the electron. For some frequencies in the neighborhood of resonance, the electron loses energy to the EM wave; this results in a coherent gain of the wave, or stimulated emission. The maximal EM wave amplification per pass is[4]

$$\Gamma = 8\lambda^5\ell^{-1}(\lambda_1^{-2}-\lambda_2^{-2})^2(\cos\theta + r\lambda/\ell)ieRL^3\sin^2(\pi r\ell_1/\ell)\sin^2\theta/mc^2\ell\pi^4\cos\theta, \qquad (4)$$

where $R = 377\ \Omega$ is the vacuum impedance. The required current density i is as high as $\sim 10^{10}-10^{11}$ A/cm^2 in order to obtain a considerable gain (~5%), in the range ~ 100Å. With the mirrors situated outside the superlattice to form a Fabry-Perot resonator to provide feedback, the system becomes a short-wave laser. The amplifiers and lasers based on the proposed principle should work in the very short pulse regime of operation, with the duration of current pulse being determined by the heating, ionization,[4] diffusion of absorbed electrons, etc. In our most recent research[7] we also developed a quantum theory of both stimulated and spontaneous emission; in classical limit the results of this theory agrees with the previously obtained results based on classical theory of the resonant transient radiation.[2–4]

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