Ultimate Bistability: Hysteretic Resonance of a Slightly-Relativistic Electron

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Abstract—It was recently predicted by us that cyclotron resonance of free electrons in vacuum and conduction electrons in semiconductors may exhibit bistable and hysteretic behavior which is due to the relativistic mass-effect (or pseudo relativistic—in semiconductors). Consistent with this prediction, the hysteretic cyclotron resonance of a trapped single electron in vacuum has recently been experimentally observed by Gabrielse *et al.* A preliminary estimate shows that their experimental results are consistent with the relativistic nature of the observed hysteresis. In this paper we consider this phenomenon as ultimate bistability since it is based on the most fundamental mechanism of nonlinearity (the relativistic mass-effect), involves the interaction of an EM wave with the simplest single elementary particle, and exhibits the first known intrinsic bistability with no macroscopic optical feedback. We also show that a hysteretic resonance of a single electron based on relativistic effects is feasible also in a parabolic potential (with no magnetic field required to attain a resonance).

INTRODUCTION

OPTICAL BISTABILITY [1] is a rapidly expanding and promising field in nonlinear optics which offers both new insights in nonlinear interactions of light with matter and potentials for superfast switching devices for optical computers and optical signal processing. Therefore, the fundamental physical problems related to that phenomenon have become important as well. One of the interesting questions is: what is the ultimate physical level of bistable interaction of light with matter? Is it feasible to realize (and possibly, to exploit) the bistable interaction at the microscopic level?

Recently it was predicted by us [2] that even a slight relativistic mass-effect of a single free electron may result in large nonlinear effects such as hysteresis and bistability in cyclotron resonance under action of an electromagnetic (EM) wave. This effect may be viewed as the ultimate and fundamental one in many respects as follows:

1) it suggests the bistable interaction of an EM wave with the single simplest microscopic physical object—an electron;
2) the nonlinearity that makes the bistable interaction possible is based on one of the most fundamental physical effects—a relativistic change of electron mass;
3) it offers bistability based on the intrinsic property of an microscopic object rather than on macroscopic optical feedback in a nonlinear medium.

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Very recently, consistent with this prediction, the hysteretic (bistable) cyclotron resonance of a free electron has been discovered by Gabrielse *et al.* [3] in an experiment in which a single electron has been trapped in a Penning trap for the period of time as long as 10 months. The electron was weakly confined in a Penning trap and oscillated in a direction which is orthogonal to the cyclotron motion with a frequency that was measurably shifted in proportion to the electron's kinetic energy. Our preliminary estimate shows (see Section II) that the experimental results are consistent with some of the theoretical predictions which confirms the relativistic nature of the hysteretic effect.

In [2] it was also suggested that the analogous effect (i.e., hysteretic cyclotron resonance) can also be expected in semiconductors with a narrow energy gap between conduction and valence bands. This problem was later addressed in [4]. In semiconductors, the mass-effect required for the hysteretic resonance is attributed to the nonparabolicity of the semiconductor conduction band which causes a pseudorelativistic dependence [5], [6] of the effective mass of conduction electrons on their momentum or energy.

In both the cases (i.e., free-space electrons and conduction electrons), the hysteretic resonance is attributed to 1) the dependence of the cyclotron frequency of forced oscillations on the relativistic mass of the electron, and hence on its momentum (or kinetic energy), and 2) the presence of finite energy losses (in particular, the most fundamental losses which are due to the EM radiation of rotating electron). The fact that the frequency of cyclotron motion is decreasing as the kinetic energy of the particle increases, is well know in the theory of the cyclotron resonance at relativistic energies [7]. This fact led to the synchrotron [8] and synchro-cyclotron [9] principles of particle acceleration. The decrease of the frequency of particle rotation (which is due to the increase in the particle energy) in the synchro-cyclotron is compensated by a corresponding decrease of the frequency of the applied voltage with time.

Although the effect of the relativistic change of cyclotron frequency was discussed basically in application to ultrarelativistic particles [7]–[9], it is obvious that it holds true for any energy of cyclotron motion. From an experimental standpoint, the problem was that at low kinetic energy the effect becomes very small since it is proportional to β^2 (where $\beta = v/c$) if $\beta^2 << 1$. However, with the increased accuracy and sensitivity of new measurements of single particles in the Penning trap [10], the effect of the slight relativistic change of cyclotron frequency has recently been observed [11] also for the kinetic energies of electron as low as a few volts. On the other hand, it was demonstrated [2] that even such a slight relativistic change of cyclotron frequency may result in strong hysteretic jumps in cyclotron resonance at low excitation energies. The additional factor required for the hysteresis to occur is the presence of energy losses, and therefore the finite linewidth of the cyclotron resonance. For example, the most pronounced jump (namely that one from the higher excitation branch to the lower one) is observed when the swept frequency of the driving EM field passes the relativistically shifted resonant point at which the kinetic energy of an excited electron reaches the maximal possible level (determined by the energy losses); see Fig. 1(a). The jump then occurs because the excitation cannot be supported at that high level anymore since when the detuning further increases, the relativistic shift of the resonance frequency rapidly decreases, which results in sudden switching from an on-resonant state to an off-resonant one. The critical condition for hysteresis (and therefore, bistability) to occur is that the relativistic shift of the cyclotron frequency must be sufficiently greater than the linewidth of the resonance. Since the linewidth of the cyclotron resonance in vacuum is determined only by the resonant frequency, the electric charge, and the rest mass of the electron (see below, Section I), the critical characteristics of this effect are also of fundamental nature. The hysteretic cyclotron (as well as noncyclotron, see Section III) resonance of electron may be therefore considered as the "ultimate" example of hysteresis in a classic nonlinear oscillator.

The hysteresis in a nonlinear oscillator based on a relativistic electron resembles the analogous effects in the oscillators with anharmonic (i.e., nonparabolic) potential [12] (in particular, the so-called Duffing oscillator, e.g., pendulum) or in so-called nonlinear parametric systems [13]. One has to note though that there are some considerable differences between nonlinear resonance of relativistic electrons and of oscillators with nonlinear potential. First, the latter oscillators can demonstrate hysteretic effects even when they have just one degree of freedom (i.e., when they are described by a scalar nonlinear differential equation of second order, or by the set of two equations of first order), whereas the cyclotron resonance of an electron in a Penning trap is to be described at least by four equations of first order (see Section II below). This corresponds to the fact that at least two degrees of freedom must be taken into consideration. Therefore, under the action of a sufficiently strong driving wave, the motion of the electron may demonstrate quite different kinds of instabilities and chaotic behavior as compared to the one-dimensional nonlinear oscillator. Second, the very nature of the relativistic nonlinearity is different from systems with a nonlinear potential. This is due to the fact that in the relativistic case the nonlinear terms are the product of speed and coordinate whereas in the "anharmonic potential" case they are only functions of the coordinate. For example, in the Duffing oscillator the first nonlinear term is proportional to x^3 (where x is the coordinate), whereas in the simplest case of noncyclotron resonance of an electron (see Section III below), the nonlinear term is proportional to $x\dot{x}^2$. It turns out, however, that the truncated equations of motion of an electron (and especially, the steady-state characteristics [2]) are very close [14] to those for systems with nonlinear potentials with the same degrees of freedom. However, this is valid only for the so-called main resonance, i.e., that one for which the driving frequency ω is near the linear eigenfrequency of the system ω_o. For other nonlinear resonances, in particular, for a subharmonic resonance of the third order (i.e., when $\omega \approx 3\omega_o$), the behavior of those two kinds of systems could be quite different.

The resemblance of the system in consideration to a nonlinear oscillator must not obscure the fact that, from the electrodynamic standpoint, the bistable (hysteretic) resonance of a single electron (as well as conduction electrons in solid state) is different from other kinds of optical bistability. One of the important features of this effect is that it is based on the intrinsic properties of the microscopic components, not on the macroscopic optical feedback. This differs from conventional mechanisms of optical bistability [1] in that so far they have been based on macroscopic nonlinear properties of the media. Indeed, a nonlinear change in macroscopic susceptibility under action of the strong EM wave provides a dramatic change in the condition for optical propagation of this wave under various special circumstances which, in turn, leads again to the change in the susceptibility. This so-called optical feedback in nonlinear macrosystems results in the existence of multistable (in particular, bistable) steady states. No such optical feedback exists in the hysteretic electron resonance. One of the consequences of this fact is that those effects exhibit also cavityless (or resonator-free) bistability [15]. Recently, some new mechanisms of optical bistability based on the intrinsic bistability which resemble bistable cyclotron resonance (in the sense that they are attributed to the nonlinear shift of some resonant frequency of material) have been proposed [16], [17] and experimentally observed [16].

In this paper we discuss the general equation of the relativistic electron motion (Section I), and briefly review the theory of hysteretic (bistable) cyclotron resonance of a single free electron and conduction electrons in semiconductors (Section II). We show also that in fact the presence of a magnetic field (which is required to attain a cyclotron resonance) is not a necessary condition for a hysteretic effect to occur; one may attain a bistability based on relativistic mass-effect even in any one-dimensional oscillator having parabolic potential well (Section III).

I. Equation of Motion

We assume that the electron (with an electrical charge e and a rest mass m_o) moves under the combined action of the static magnetic field $\vec{H}_o$, static electric field $\vec{g}$, and driving electromagnetic (EM) wave $\vec{E}_{\text{in}}(t)$. We treat this problem classically. The equation of motion for the electon moving with arbitrary velocity v is [18]

$$d(m\vec{v})/dt = (e/c)\ \vec{v} \times \vec{H}_\Sigma + e\vec{E}_\Sigma + \vec{F}_l \quad (1.1)$$

$$m = m_o(1 - |\vec{v}|^2/c^2)^{-1/2} \quad (1.2)$$

where $\vec{H}_\Sigma$ is the total magnetic field (including the EM wave component $\vec{H}_{\text{EM}}$, i.e., $\vec{H}_\Sigma = \vec{H}_o + \vec{H}_{\text{EM}}$), $\vec{E}_\Sigma = \vec{E}_{\text{in}} + \vec{g}$ is the total electric field, and the term $\vec{F}_l$ represents energy losses of the electron. In the ultimate case in which the losses are caused by EM radiation of the rotating electron (and $|v| << c$) this term can be written as [18]

$$\vec{F}_l = (2e^2/3c^3)\ d^2\vec{v}/dt^2. \quad (1.3)$$

In the general case the losses may be much larger and caused by various factors. The force is then proportional to the velocity of the electron, e.g., $\vec{F}_l = -\gamma m_o \omega_0 \vec{v}$, where γ is the dimensionless bandwidth of cyclotron resonance. The radiation losses can also be represented by this formula, since one can assume [18] that $d^2\vec{v}/dt^2 \approx -\omega_o^2\vec{v}$, which yields

$$\gamma_{\text{rad}} = \frac{2e^2\omega_o}{3m_oc^3} = \frac{2}{3} r_e k_o << 1 \quad (1.4)$$

where $r_e = e^2/m_oc^2 = 2.8 \times 10^{-13}$ cm is an electron radius and $k_o = \omega_0/c$ is a resonance wave number. If the EM field is a plane wave, then $\vec{H}_{\text{EM}} = \vec{k} \times \vec{E}_{\text{in}}/\omega$ (where ω is a frequency of EM field, and $k = \omega/c$), and (1) can be written in the form

$$d(m\vec{v})/dt + \gamma m_o\omega_0\ \vec{v} = e\left(\vec{E}_{\text{in}} + \frac{1}{c}\ \vec{v} \times \vec{H}_o\right) + e\left(\frac{\vec{v}}{\omega} \times [\vec{k} \times \vec{E}_{\text{in}}] - \vec{g}\right) \quad (1.5)$$

where the term $e\vec{v} \times [\vec{k} \times \vec{E}_{\text{in}}]/\omega$ is a radiation force applied to the electron. Usually this term is neglected (see, e.g., [14]), except in [19] in which, however, losses and other possible forces like potential $\vec{g}$ were not considered. However, all of these interactions become important [2] when considering excited steady states (and multistability) of the electron under the action of the sufficiently intense EM wave. Equation (1.5) is the general equation governing the nonlinear resonance of electron. In the case of slight relativism ($v^2 << c^2$), the mass m in the left-hand side of (1.5) is written as $m \approx m_o(1 + v^2/2c^2)$, where the term $v^2/2c^2$ is responsible for small nonlinear relativistic effects.

II. Hysteretic (Bistable) Cyclotron Resonance of Electrons in Vacuum and Solids

Consider the case in which the static magnetic field H_o is sufficiently strong ($H_o >> g$, E_{in}) and the plane EM wave $\vec{E}_{\text{in}}$ (with amplitude E) propagates along the z axis parallel to $\vec{H}_o$. The field H_o provides a cyclotron resonance with the unperturbed frequency $\omega_o = eH_o/m_oc$. We also assume that a small static electric field $\vec{g}(z)$ is applied along the z axis to arrange a trapping [10] of the electron and to compensate a radiation force. We introduce dimensionless notations

$$\vec{\beta} = \frac{\vec{v}}{c};\quad \vec{\mu} = \frac{e\vec{E}_{\text{in}}}{m_oc^2}\frac{1}{k_o};\quad \rho(z) = \frac{eg(z)}{m_oc^2} \cdot \frac{1}{k_o}. \quad (2.1)$$

All these variables (as well as γ) are supposed to be very small compared to unity. We also asssume that the EM wave $\vec{E}_{\text{in}}$ is circularly polarized (which maximizes the expected effect) and rotates in the same direction as the electron, i.e., $\vec{\mu} = \mu[\hat{e}_x \sin(\omega t - kz) + \hat{e}_y \cos(\omega t - kz)]$. The required solution to (1.5) can then be written in the form

$$\vec{\beta}(t, z) = \beta[\hat{e}_x \sin(\omega t - kz + \phi) + \hat{e}_y \cos(\omega t - kz + \phi)] + \beta_z\hat{e}_z \quad (2.2)$$

where the unknown variables β, β_z, and ϕ vary little in the time ω^{-1}. In the slow-varying envelope approximation, one can write down the set of truncated first-order equations

$$\dot{\beta}/\omega_o = -\gamma\beta + \mu\cos\phi;$$

$$\dot{\phi}/\omega_o = \beta_z - (\Delta + \beta^2/2 + \mu\sin\phi/\beta) \quad (2.3)$$

$$\dot{\beta}_z/\omega_o = -\gamma\beta_z - \rho(z) + \mu\beta\cos\phi;\quad \dot{z} = c\beta_z. \quad (2.4)$$

Here $\Delta = (\omega - \omega_o)/\omega_0 << 1$; Δ is a dimensionless resonant detuning. The steady-state solution ($d/dt = 0$) is thus determined by the relationships

$$\mu^2 = \beta_s^2[\gamma^2 + (\Delta + \beta_s^2/2)^2];$$

$$\tan\phi_s = -(\Delta + \beta_s^2/2)/\gamma \quad (2.5)$$

$$(\beta_z)_s = 0;\quad \rho(z_s) = \gamma\beta_s^2 \quad (2.6)$$

where the subscript "s" labels characteristics of the steady-state regime. Under the threshold conditions

$$\mu^2 > \mu_{\text{th}}^2 \equiv (16/3\sqrt{3})\ \gamma^3,\quad \Delta < \Delta_{\text{th}} \equiv -\gamma\sqrt{3}, \quad (2.7)$$

(2.5) yields a three-valued solution for β_s (Fig. 1). At the threshold point this value is $\beta_{\text{th}}^2 = 2\gamma/\sqrt{3}$ (curve 2 in Fig. 1), and the radius of orbit is $r = \beta_{\text{th}}/k_o << \lambda_0$. The condition (2.7) warrants the existence of at least one jump (from the lower steady state branch to the uppper one in Fig. 1(a). However, in order for the hysteresis to exist (i.e., to warrant the existence of the second jump from the upper branch to the lower one in Fig. 1(a), one has to require nonzero losses, i.e., $\gamma > 0$. In the case of sufficiently strong pumping (i.e., $\mu^2 >> \mu_{\text{th}}^2$), the detuning Δ_h and the kinetic energy of the excited electron $\beta_h^2/2$ that correspond to the second jump follow from (2.5) as

$$\Delta_h \approx -\mu^2/2\gamma^2; \quad (2.8a)$$

also,

$$\Delta_h = -\beta_h^2/2. \quad (2.8b)$$

In such a case, the upper branch of the function $\beta_s^2(\Delta)$

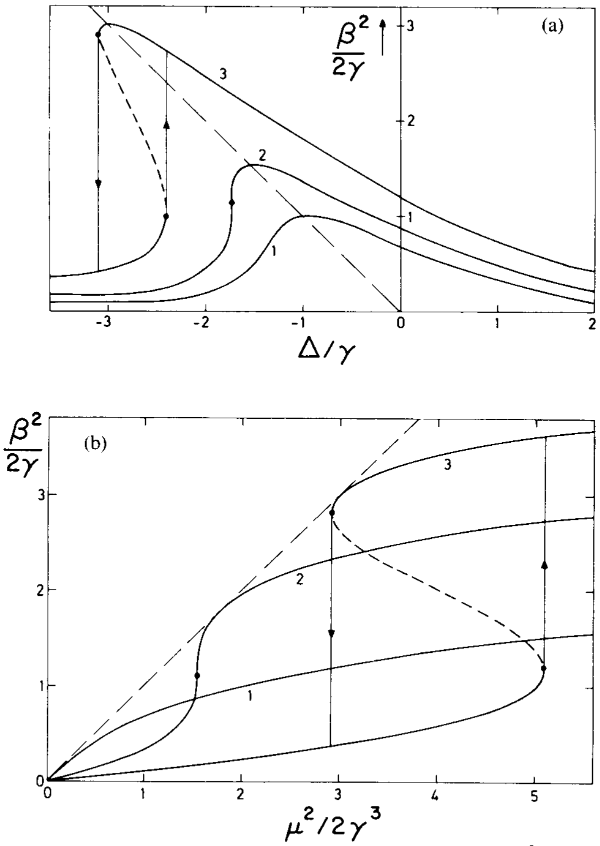

Fig. 1. The plots of normalized kinetic energy of the electron $\beta^2/2$ (a) versus normalized resonant detuning Δ/γ for various intensities of incident EM field, and (b) versus normalized incident intensity $\mu^2/2\gamma^3$ for various detunings. Curves: (a) 1, $\mu^2/2\gamma^3 = 1$; $\mu^2/2\gamma^3 = (2\sqrt{3})^3$; 3, $\mu^2/2\gamma^3 = 3$; (b) 1, $\Delta/\gamma = 0$; 2, $\Delta/\gamma = -\sqrt{3}$; 3, $\Delta/\gamma = -3$.

in the range $\Delta_h < \Delta < 0$ is approximately a linear one: $\beta_s^2/2 \approx -\Delta$. Although the experimental data in [3] are insufficient to confirm (2.8a) (since the driving field intensity at the location of the electron was not reported), they are in good agreement with (2.8b) and with the linearity in the function $\beta_s^2(\Delta)$. Indeed, using Fig. 2 in Ref. [3], one estimates $|2\Delta_h/\beta_h^2| \approx 0.92$ which is quite close to 1.0 as required by (2.8b). This confirms that the nonlinearity of the system is attributed (at least to the large degree) to the relativistic mass-effect.

In the case of the multivalued solution the examination of (2.3) and (2.4), linearized in the close vicinity of the steady-state solutions (2.5) and (2.6), shows that only those states are stable which satisfy the energy criterion $d(\beta_s^2)/d(\mu^2) > 0$ (solid branches of the curves in Fig. 1); otherwise, they become unstable (dashed branches in Fig. 1). Let us make some quantitative estimates for the threshold parameters (2.7). A magnetic field of strength $H_o = 100$ kG produces the cyclotron frequency $\omega_o = 1.7 \times 10^{12}$ s^{-1} ($\lambda_o \sim 1.07$ mm). Then in the ultimate case of radiation losses, the resonance width is $\gamma \sim 10^{-11}$, which yields the threshold field amplitude as small as $E_{th} = 1.7 \times 10^{-1}$ V/cm, and the kinetic energy as small as $\beta_s^2/2 \simeq 1.2 \times 10^{-11}$. This is, in fact, even near α^{-1} times smaller than a quantum limit of the energy of excitation which is $2\hbar\omega_o/mc^2$ (here $\alpha = e^2/\hbar c = 1/137$ is the fine-structure constant). Therefore, in the close vicinity of the threshold (2.7), only the quantum approach can give an adequate description of the phenomenon, whereas for sufficiently strong driving field ($\mu >> \mu_{th}$) the classical results (and, in particular, hysteretic jumps) remain valid. The experimental data [3] obviously represent the hysteretic effect in the classical limit. The further improvement of the stability and bandwidth of the source of the driving radiation may hopefully bring the experiment to the threshold measurements and therefore, to the quantum limit of the hysteresis effect.

Although the above discussion was dealing with a single free electron, the hysteretic cyclotron resonance may also exist for conduction electrons in solids, in particular in semiconductors [4]. This effect is feasible due to the nonparabolicity of the semiconductor conduction band which causes a pseudorelativistic behavior [5], [6] of the effective mass of conduction electrons in the narrow-gap semiconductors such as, e.g., InSb. Indeed, in narrow-gap semiconductors which can be described by the Kane two-band model [5] with isotropic nonparabolic bands, the conduction band energy W (which is an analog of kinetic energy) can be written as

$$W(p) = \sqrt{m_o^{*2}v_o^4 + p^2v_o^2} \tag{2.9}$$

where $\vec{p}$ is the momentum of the conduction electron, m_o^* is its effective mass at the bottom of the conduction band, $v_o = \sqrt{W_G/2m_o^*}$ is some characteristic speed, and W_G is the band gap (the energy W in (2.9) is measured with respect to the middle of the gap). The velocity $\vec{v}$ of the conduction electron is given by [20] $\vec{v}(\vec{p}) = \partial W(p)/\partial\vec{p}$, which yields

$$\vec{p} = m_o^*\vec{v}/\sqrt{1 - v^2/v_o^2}; \quad \vec{v} = \vec{p}/m_o^*\sqrt{1 + p^2/p_o^2} \tag{2.10}$$

where $p_o \equiv m_o^*v_o = \sqrt{W_G m_o^*/2}$ is some characteristic momentum. One can see that relations among W, v, and p are completely relativistic, with v_o playing a role as an "effective speed of light," and $W_G/2$ as an "effective rest energy" of the electron. For InSb, $W_G = 0.24$ eV, $m_o^* = 0.014\ m_o$ so that $v_o \sim 1.15 \times 10^8$ cm/s $<< c$. The fact that in semiconductors $m_o^* << m_o$ and $v_o << c$ provides new interesting opportunities: 1) the smallness of the effective mass m_o^* results in a considerable increase of the cyclotron frequency (up to 70–80 times) for the same magnetic field and therefore, helps bring the experiment to the infrared range, and 2) the smallness of v_o allows one to attain a fairly low critical pumping intensity (although still much greater than in vacuum) for observation of the effect even taking into consideration the much faster relaxation in semiconductors. For example, in order to use a radiation of CO_2 laser with $\lambda_o \sim 10.6$ μm, the magnetic field $H_o \sim 140$ kG is required to observe a cyclotron resonance in InSb. Making a reasonable assumption of $\gamma \sim 10^{-2}$, one estimates [4] the critical power required to observe a hysteretic resonance of the order ~ 240 W/cm^2.

It is interesting to note that a seemingly similar effect based on the electron-spin resonant shift in metals if there is an appreciable degree of nuclear polarization was briefly mentioned in [21] (although it is suggested in [21] that the resonant line shifts toward higher frequencies as opposed to the relativistic or pseudorelativistic mechanisms of nonlinearity discussed above). It was also theoretically shown that another hysteretic effect in self-consistent magnetization based on the inverse Faraday effect is feasible in semiconductors [22] which critically depends on concentration of conduction electrons (or carriers) with constant effective mass; this apparently suggests a phase transition kind of hysteresis (albeit, not a pseudorelativistic one).

III. Bistable Noncyclotron Resonance of a Slightly-Relativistic Electron

In the previous sections, the bistable effect was discussed which is based on rotation of a slightly-relativistic electron in a magnetic field (i.e., on a nonlinear cyclotron effect). However, it is obvious that the presence of a magnetic field is not a necessary condition; the only factors substantial for the bistable resonant excitation to exist are the presence of nonlinearity (in our case—relativistic mass-effect) and a sufficiently sharp resonance. The latter can be provided by any potential well. In the simplest and probably most characteristic case it is a parabolic potential well. This would correspond to a conventional linear oscillator had the electron mass not varied due to relativistic effect. We assume that the electron oscillates along the x axis, and is excited by the driving periodic electric field $E_{\text{in}} = E \sin \omega t$, which is directed along the same axis. Thus, in (1.5) one has to assume $H_o = 0$, and

$$\vec{g} = \frac{m_o}{e} \omega_o^2 x \hat{e}_x \tag{3.1}$$

where ω_o^2 is an eigenfrequency of a respective linear oscillator. We assume as usual that the motion is only slightly relativistic (i.e., $\dot{x}^2 << c^2$). In such an idealized case, the motion of the electron is governed by the equation

$$\ddot{x} + \gamma\omega_o\dot{x} + x\omega_o^2(1 - \dot{x}^2/2c^2) = \frac{eE}{m_o} \sin \omega t \tag{3.2}$$

This equation is very similar to the Duffing equation except that instead of a nonlinear term x^3 (which would correspond to the anharmonic potential) it has the term $x\dot{x}^2$. As mentioned above, this does not result in a significant difference in the truncated dynamic equations nor in characteristics of steady-states for the main resonance (i.e., in the case when $\omega \approx \omega_o$). However, the difference will become substantial for higher order nonlinear resonances, e.g., for generation of the third harmonic (i.e., when $3\omega \approx \omega_o$) or the third subharmonic (i.e., when $\omega \approx 3\omega_o$). We use the same envelope approximation as in Section II and look for the forced solution of (3.2) in the form

$$x(t) \approx v_m(t)\, \omega^{-1} \sin (\omega t + \phi) \tag{3.3}$$

where the maximal speed of electron v_m and its phase ϕ are slow varying functions of time. We introduce again a dimensionless variables

$$\beta_m = \frac{v_m}{c}; \quad \Delta = \frac{\omega - \omega_o}{\omega_o}, \quad \mu = \frac{eE}{m_o c^2} \cdot \frac{1}{k_o} \tag{3.4}$$

and assume that all of them are much smaller than unity. Then, in the same fashion as in Section II, one gets truncated equations for their dynamics

$$\dot{\beta}_m/\omega_o = -\gamma\beta_m/2 + \mu \cos \phi/2 \tag{3.5}$$

$$\dot{\phi}/\omega_o = -(\Delta + \beta_m^2/32 + \mu \sin \phi/2\beta_m) \tag{3.6}$$

which (with modified coefficients and $\beta_z = 0$) reproduce (2.3). The steady-state regimes ($d/dt = 0$) follow immediately:

$$\mu^2 = 4\beta_m^2[\gamma^2/4 + (\Delta + \beta_m^2/32)^2] \tag{3.7}$$

$$\tan \phi = -2(\Delta + \beta^2/32)/\gamma \tag{3.8}$$

which again reproduces (2.5) with modified coefficients. By introduction $\tilde{\mu} = \mu/8$, $\tilde{\gamma} = \gamma/2$, and $\tilde{\beta}_s = \beta_m/4$, (3.7) and (3.8) are reduced exactly to (2.5) for $\tilde{\mu}$ and $\tilde{\beta}_s$. Therefore, taking into consideration (2.7) and (3.4), the threshold conditions required in order to observe bistability of excitation of a slightly-relativistic oscillator (3.2) are

$$|eE| > 16\, k_o m_o c^2 (\gamma/2\sqrt{3})^{3/2}; \quad \omega - \omega_o > \omega_o \gamma \sqrt{3/2}. \tag{3.9}$$

If the damping rate γ is again due to radiation of EM wave by the electron (1.4), the required threshold intensity of the driving field is of the same order of magnitude as that one discussed in Section II for the bistable cyclotron resonance in vacuum.

Conclusion

In conclusion, we demonstrated the feasiblity of hysteretic behavior and bistability of the cyclotron resonance of free electrons in vacuum and of conduction electrons in semiconductors under the action of sufficiently strong quasiresonant driving radiation. We showed also that the same effect must be peculiar to a conventional (i.e., noncyclotron) resonance of a slightly-relativistic electron in a harmonic potential well. Further research should involve quantum as well as kinetic theory of the phenomenon. Even far from the onset of the hysteresis, the action and the strong pumping should cause a dramatic change in the location and the shape of the resonant curve of cyclotron resonance, in particular, the shape of the resonant curve as a function of the frequency (or magnetic field) should become drastically asymmetrical. This effect may provide a new experimental method to measure the nonparabolicity of the conduction band in semiconductors, effective mass, nonlinear relaxation, etc. Probably the most attractive and fundamental feature of all these effects is that for the first time they provide a unique opportunity to study hysteretic and bistable phenomena at a quantum level which is especially true in the case of a single trapped electron in a free space.

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A. E. Kaplan, for a photograph and biography, see this issue, p. 1536.