Reprinted from Nature, Vol. 317, No. 6037, pp.476-477, 10 October 1985

Electron trapping

Hysteretic relativistic resonance of a single electron

from A. E. Kaplan

A SINGLE electron, trapped in a volume of less than one cubic millimetre for an unprecedented 10-month term of confinement, has allowed the first observation of a predicted fundamental hysteretic effect[1]. It is due to the relativistic increase — albeit very slight — of mass with velocity. In a recent issue of *Physical Review Letters*, G. Gabrielse and others at the University of Washington, Seattle, report that the cyclotron resonance of a single electron trapped in a Penning trap was hysteretic[2]. The striking fact is that the relativistic change required[1,2] is very small; since the excitation energy of the electron was usually less than 1 eV, the mass change was of the order of only a millionth of the electron rest mass, m_0.

Why does the hysteresis occur? In a magnetic field B, an electron rotates with a cyclotron frequency $\omega_c = eB/mc$ (where e is the electrical charge of an electron and c is the speed of light), with its mass m relativistically dependent on its velocity v. Therefore, when the electron is excited by the resonant driving field (with its driving frequency ω close to ω_c), the cyclotron frequency decreases. Suppose now that the initial driving frequency of the external field is larger than the non-relativistic cyclotron frequency ω_0 (corresponding to m_0) and is slowly swept downwards, towards the exact resonance. As the driving frequency approaches the resonant frequency, the electron gets excited and its dimensionless orbit speed $\beta = \mathrm{v}/c$ increases. If relativistic effects could be ignored, β would show the familiar Lorentzian resonant response. If, however, the amplitude E of the driving field is sufficiently large, the relativistic redshift of the cyclotron frequency yields a drastically different response, which is depicted by the solid curve in the figure. As the driving frequency ω is swept downward, dimensionless speed β and kinetic energy $\beta^2/2$ increase (upper branch of the solid curve in the figure), and the cyclotron frequency ω_c is shifted further away from the unperturbed resonant frequency ω_0 as if it is running away from the driving frequency.

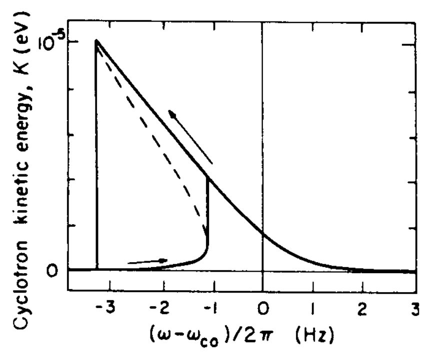
This continues until the maximum level of excitation β_m is reached at the point of exact resonance, where $\omega = \omega_c = \omega_0 (1 - \beta_m^2/2)$. The maximum level of excitation β_m is proportional to the amplitude of a driving field and inversely proportional to the frequency width of resonance Γ, which is determined by the radiation dissipation of energy. If ω is swept further downwards, the speed of the excited electron begins to diminish, and the effective cyclotron frequency starts to run back. For this reason, very shortly after the driving frequency passes the point of exact resonance, the excitation can no longer be supported at the high level of the resonant state and suddenly jumps to the off-resonant state. If the driving frequency is then swept backwards, the other jump (from lower to upper branch in the figure) occurs when ω is very close to ω_0.

This cycle forms a relativistic hysteresis within the range in which the electron can have two possible steady states of excitation — the simplest and probably the most fundamental bistable system. (The middle branch in the figure corresponds to an unstable regime[1], and cannot be observed.) For the hysteresis to occur, the maximum relativistic shift of the cyclotron frequency, $\omega_0 \beta_m^2/2$ must be sufficiently greater than (but of the same order of magnitude as) the width of the resonance, Γ. This condition yields[1] an unexpectedly low critical energy of excitation for the electron that is nearly α^{-1} times smaller than a quantum limit $\hbar\omega$ ($\alpha = e^2/\hbar c = 1/137$ is the fine-structure constant). Therefore, in the close vicinity of the threshold, only the quantum approach can give an adequate description of the phenomenon whereas, for a sufficiently strong driving field, the classical results (in particular, hysteretic jumps) remain valid. For the experimental situation the critical kinetic energy of excitation is of the order of 10^{-5} eV but the actual excitation used by Gabrielse *et al.* was a few orders of magnitude greater and so the experimental data obviously represent the classical limit.

The fact that the frequency of cyclotron motion decreases as kinetic energy of the particle increases is well known in the theory of the application of synchrotrons and synchro-cyclotrons to the study of ultra-relativistic particles. For Gabrielse *et al.* to observe cyclotron hysteresis it was necessary to work at energies as low as a few electron volts, which only became possible with improved measurements of single particles in the Penning trap[3]. (A single-atom trapping technique has simultaneously been developed based on laser cooling[4].) The Penning trap is essentially a positive ring and two negative cap electrodes kept at liquid helium temperature in the core of a 60-kG superconducting magnet. The electron was prevented from escaping by confinement in a relatively weak quadruple potential of the trap. The potential of the trap was also used to measure a relativistic mass increase through the axial frequency shift.

Apart from one basic difference, the observed hysteresis very much resembles the so-called vibration hysteresis in anharmonic oscillators with nonparabolic potential[5], in particular the Duffing oscillator in which the shift of resonant frequency is due to the higher-order terms of potential. The difference is that both factors responsible for the single-electron hysteretic effect are of fundamental nature: the relativistic change of mass (and hence cyclotron frequency), and a damping factor Γ which depends only on basic constants.

That is why the observed effect is so important. It may be viewed as the ultimate example of optical bistability (reviewed in ref. 6), which is a rapidly expanding and promising field in nonlinear



Resonant line shape (solid curve) of a slightly relativistic cyclotron resonance. The localization of the electron's steady-state kinetic energy on either upper or lower branch of the solid curve depends upon the direction (arrows) in which the driving frequency is swept through resonance and includes discontinuous jumps (broken line).

optics that offers new insights into the interaction of light with matter as well as the potential for superfast switching devices for optical computers and signal processing. The observed effect is also the first example of an intrinsic optical bistability; all other optical bistable effects depend upon a change in the macroscopic state of a nonlinear medium (the so-called optical feedback).

What of the future? The University of Washington group is planning to measure the magnetic moment of an electron with much improved accuracy using hysteretic resonance. Obviously, it will be of great interest to study the hysteresis in the vicinity of its threshold since this will provide a unique opportunity to study a quantum limit of bistable oscillators. Our group is working on this kind of quantum theory of the phenomenon as well as new multiphoton methods of optical excitation of single-electron cyclotron resonance. As far as the classical limit is concerned, one may envisage that, under the action of a sufficiently strong driving field, the upper branch of the hysteretic curve will become unstable, resulting in the chaotic motion of an electron similar to chaos in other nonlinear oscillators. Finally, it is worth noting that a similar hysteretic behaviour might be observed for electrons in semiconductors[7], in which case the relativistic-like change of the effective mass of conduction electrons would be attributable to the non-parabolic nature of the semiconductor conduction band. □

1. Kaplan, A. E. *Phys. Rev. Lett.* **48**, 138 (1982).
2. Gabrielse, G., Dehmelt, H. & Kells, W. *Phys. Rev. Lett.* **54**, 537 (1985).
3. Gabrielse, G. & Dehmelt, H. in *Precision Measurements and Fundamental Constants II* (ed. Taylor B. N. & Phillips, W. D.) (US NBS Spec. Publ. 617, Washington, DC, 1984).
4. Petley, B. W. *Nature News and Views* **315**, 716 (1985).
5. Landau, L. D. & Lifshitz, E. M. *Mechanics* (Pergamon, New York, 1976).
6. Abraham, E. & Smith, D. *Rep. Prog. Phys.* **45**, 815 (1982).
7. Kaplan, A. E. & Elci, A. *Phys. Rev.* **B29**, 820 (1984).

A. E. Kaplan is Professor of Electrical Engineering, Purdue University, West Lafayette, Indiana 47907, USA.

Printed in Great Britain by Macmillan Production Ltd., Basingstoke, Hampshire