Relativistic Nonlinear Optics of a Single Cyclotron Electron

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Because of relativistic nonlinear effects large-amplitude cyclotron motion of a free electron can be excited by two laser beams with frequencies much higher than the cyclotron frequency Ω. The laser frequencies can differ either by Ω (three-photon resonance) or 2Ω (four-photon resonance). The excited cyclotron motion displays a hysteretic resonance based solely on the relativistic-mass effect. To observe three-photon hysteretic excitation, a He-Ne laser with a few microwatts of power is sufficient.

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The relativistic effects experienced by a single elementary particle (e.g., electron) oscillating under the action of an electromagnetic (EM) field are probably the most fundamental mechanisms of nonlinear interaction of light with matter. It was shown earlier[1] that because of small relativistic changes of mass, a single electron can exhibit pronounced hysteretic cyclotron resonance when the frequency of the driving EM field is near the cyclotron frequency (which usually, for the presently available magnetic fields, is in the microwave range). Because of very low radiative losses, the relativistic change of mass to which this effect is attributable may be as small as $\sim 10^{-10}$–10^{-6}. Consistent with this prediction, this effect has recently been observed in an experiment[2] with a single electron trapped for ten months in a Penning trap.

In this Letter I show that because of the relativistic effects still another group of nonlinear optical effects is feasible: large excitation of (microwave) cyclotron motion of an electron by two *optical* waves with their respective frequencies ω_1 and ω_2 ($\omega_1 > \omega_2$) being much higher than the cyclotron frequency Ω (ω_1, $\omega_2 \gg \Omega$) and differing by either Ω or 2Ω. Hence, these effects may be regarded as three-photon and four-photon interactions, respectively. These multiphoton effects may become a starting point for an entire new field that may be described as relativistic nonlinear optics of a single electron (or of single particles, in general). They also offer a new method of excitation of the cyclotron motion which may prove more advantageous compared to the conventional methods utilizing either microwave or rf oscillators. Indeed, since the optical frequencies ω_1 and ω_2 can be provided by two modes of the same laser, they allow for easily tunable control over the difference frequency $(\omega_1 - \omega_2)$. The power of the laser light required to obtain the cyclotron excitation is sufficiently low to allow for the use of lasers in a cw or quasi-cw regime. The proposed effects may also be used for particle acceleration; even in a simple Penning trap, a kinetic energy of an excited electron as high as a few megaelectronvolts may be obtained. All of these effects exhibit a relativistic hysteresis, similar in nature to that of a cyclotron resonance which occurs at the main frequency,[1–3] when $\omega = \Omega \simeq \Omega_0$. For four-photon resonance, the excited electron can have two possible phases of cyclotron excitation (which differ by π); which phase is excited depends on the initial conditions. This may be regarded as a manifestation of a new type of optical bistability which I call phase bistability (i.e., based on bistability of the phase of oscillation rather than on bistability of its amplitude).

Consider a single free electron in a homogeneous magnetic field $\mathbf{H}_0$ which provides a cyclotron resonance with the initial frequency $\Omega_0 = eH_0/m_0c$. The electron is illuminated by the optical field which may, in general, consist of any number of plane waves $\mathbf{E}_j(\omega_j t - \mathbf{k}_j \cdot \mathbf{r})$, where ω_j and $\mathbf{k}_j$ are, respectively, the frequencies and wave vectors of the fields. We treat the problem classically; the motion of an electron with an arbitrary momentum $\mathbf{p} = m_0\gamma\mathbf{v}$ is then governed by the Lorentz equation (with an additional damping term[4]):

$$\frac{d\mathbf{p}}{dt} + \Gamma\gamma\Omega_0\mathbf{p} = e\sum_j \mathbf{E}_j + \frac{e}{\gamma m_0 c}\mathbf{p}\times\sum_j\left[\frac{\mathbf{k}_j\times\mathbf{E}_j}{k_j}\right] + \frac{e}{\gamma m_0 c}[\mathbf{p}\times\mathbf{H}_0], \tag{1}$$

where $\gamma = (1 + p^2/m_0^2c^2)^{1/2}$ and Γ is a damping parameter due to cyclotron radiation, $\Gamma = 2e^2\Omega_0/3m_0^3 \ll 1$. As distinct from previous work[1–3] we do not restrict ourselves to low excitation energies; hence γ can be significantly greater than unity. In such a case the energy losses described by the second term on the left-hand side of Eq. (1) increase proportionally to $\gamma\mathbf{p}$ (rather than to just $\mathbf{p}$ as in the slightly relativistic case[14]). The second term on the right-hand side in Eq. (1) is the Lorentz force of the incident EM wave; it is attributable to the magnetic field of the EM wave,

$\mathbf{H}_j = [\mathbf{k}_j \times \mathbf{E}_j]/k_j$. We introduce a dimensionless momentum $\boldsymbol{\rho}$, fields $\mathbf{f}_j$, unit vector of magnetic field $\mathbf{h}$, and unit wave vectors $\mathbf{q}_j$ as follows: $\boldsymbol{\rho} = \mathbf{p}/m_0 c$, $\mathbf{f}_j = \mathbf{E}_j/H_0 = e\mathbf{E}_j/m_0 c\,\Omega_0$, $\mathbf{h} = \mathbf{H}_0/H_0$, and $\mathbf{q}_j = \mathbf{k}_j/k_j$; we assume that $\boldsymbol{\rho}$ can be written in the form

$$\boldsymbol{\rho} = \boldsymbol{\rho}_c(t) + \boldsymbol{\rho}_{\rm nc}^{(1)} + \boldsymbol{\rho}_{\rm nc}^{(2)} + \dots, \quad (2)$$

where $\boldsymbol{\rho}_c$ is a "cyclotron" component of momentum describing a pure rotation of the electron around some fixed center ($\mathbf{r} = 0$) with the frequency $\Omega \simeq \Omega_0/\gamma_c$; $\boldsymbol{\rho}_c$ is orthogonal to $\mathbf{H}_0$. The various orders of "noncyclotron" components $\boldsymbol{\rho}_{\rm nc}$ include oscillations with all the other, nonresonant, frequencies and may have any orientation. The cyclotron component $\boldsymbol{\rho}_c$ is determined by the equation

$$\Omega_0^{-1}(d\boldsymbol{\rho}_c/dt) - \gamma_c^{-1}[\boldsymbol{\rho}_c \times \mathbf{h}] + \Gamma\gamma_c\boldsymbol{\rho}_c = \mathbf{F}_c^{(1)}(t) + \mathbf{F}_c^{(2)}(t) + \dots, \quad (3)$$

where $\gamma_c = (1 + |\boldsymbol{\rho}_c|^2)^{1/2} = (1 + \rho_c^2)^{1/2}$; $\mathbf{F}^{(s)}$ are nonlinear forces of different orders s; the subscript c in $\mathbf{F}_c^{(s)}$ labels those components of these forces that oscillate with the cyclotron frequency Ω and are orthogonal to $\mathbf{H}_0$. In Eqs. (2) and (3), $\boldsymbol{\rho}_{\rm nc}^{(s)}$ and $\mathbf{F}^{(s)}$ are defined as

$$\mathbf{F}^{(1)} = \sum_j \mathbf{f}_j(\omega_j t - \mathbf{k}_j \cdot \mathbf{r}_c(t)) + \gamma_c^{-1}\boldsymbol{\rho}_c \times \sum_j [\mathbf{q}_j \times \mathbf{f}_j] \quad (s = 1), \quad (4)$$

$$\Omega_0^{-1}(d\boldsymbol{\rho}_{\rm nc}^{(s)}/dt) - \gamma_c^{-1}[\boldsymbol{\rho}_{\rm nc}^{(s)} \times \mathbf{h}] + \gamma_c^{-3}(\boldsymbol{\rho}_{\rm nc}^{(s)} \cdot \boldsymbol{\rho}_c)[\boldsymbol{\rho}_c \times \mathbf{h}] = \mathbf{F}^{(s)} - \mathbf{F}_c^{(s)} \quad (s > 0), \quad (5)$$

$$\mathbf{F}^{(s)} = \mathbf{F}_{\rm D}^{(s)} + \mathbf{F}_{\rm L}^{(s)} + \mathbf{F}_R^{(s)} \quad (s > 1), \quad (6)$$

where

$$\mathbf{r}_c = c\gamma_c^{-1}\int \boldsymbol{\rho}_c\,dt = -c(\Omega\gamma_c)^{-1}[\boldsymbol{\rho}_c \times \mathbf{h}].$$

Each of the sth-order forces $\mathbf{F}_{\rm D}^{(s)}$, $\mathbf{F}_{\rm L}^{(s)}$, and $\mathbf{F}_R^{(s)}$ is defined as a sum of all terms of sth order in $\mathbf{f}_j$ originating, respectively, from the first, second, and third terms on the right-hand side of Eq. (1), respectively, in which all the lower-order terms of $\boldsymbol{\rho}$ in Eq. (2) (the highest of which is $\boldsymbol{\rho}_{\rm nc}^{(s-1)}$) are taken into account; note that $\boldsymbol{\rho}_{\rm nc}^{(s)}$ is of sth order in $\mathbf{f}_j$. The force $\mathbf{F}^{(2)}$ is given as follows:

$$\mathbf{F}^{(2)} = \sum_j (\mathbf{q}_j \cdot \mathbf{r}_{\rm nc}^{(1)})\partial\mathbf{f}_j/\partial(\mathbf{q}_j \cdot \mathbf{r}_{\rm nc}) + \gamma_c^{-1}\boldsymbol{\rho}_{\rm nc}^{(1)} \times \sum_j [\mathbf{q}_j \times \mathbf{f}_j]$$
$$- \gamma_c^{-3}\{\boldsymbol{\rho}_{\rm nc}^{(1)}(\boldsymbol{\rho}_n \cdot \boldsymbol{\rho}_{\rm nc}^{(1)}) + (\boldsymbol{\rho}_c/2)[(\boldsymbol{\rho}_{\rm nc}^{(1)})^2 - 3\gamma_c^{-2}(\boldsymbol{\rho}_c \cdot \boldsymbol{\rho}_{\rm nc}^{(1)})^2]\} \times \mathbf{h}, \quad \mathbf{r}_{\rm nc} = c\gamma_c^{-1}\int \boldsymbol{\rho}_{\rm nc}\,dt. \quad (7)$$

In Eq. (6) *three* main mechanisms of nonlinear interaction are distinguished, each of which is related to the respective term in Eq. (1). The spatial oscillations of the electron make it see the phases $\mathbf{k}_j \cdot \mathbf{r}$ of the incident fields $\mathbf{f}_j(\omega_j t - \mathbf{k}_j \cdot \mathbf{r})$ [the first term on the right-hand side of Eq. (1)] rapidly modulated since $\mathbf{r} = c\gamma^{-1}\int \boldsymbol{\rho}\,dt$. This modulation is due to the Doppler effect; hence the designation "Doppler" nonlinear mechanism, $\mathbf{F}_{\rm D}$. The Lorentz force [the second term on the right-hand side of Eq. (1)] gives rise to components with combination frequencies; hence, the designation "Lorentz" nonlinear mechanism, $\mathbf{F}_{\rm L}$. Finally, there is a relativistic-mass effect due to $\gamma^{-1} = [1 + (\boldsymbol{\rho})^2]^{-1/2}$ in the last, cyclotron, term on the right-hand side of Eq. (1); hence, the designation "relativistic" nonlinear mechanism, $\mathbf{F}_R$. Contributions from all these three mechanisms can be of the same order of magnitude. In general, none of them can be neglected; however, for particular propagation and polarization configurations some of them may dominate. It is worth emphasizing, though, that once the cyclotron motion is excited, it is only the relativistic-mass effect [the term γ_c^{-1} on the left-hand side of Eq. (3)] that acts to limit the energy of excitation and to form a hysteretic resonance.

The hierarchical ranking in Eqs. (2)–(6) is tailored in such a way as to emphasize the order of interactions in f_j, not in (ρ_c, f_j) (since we assume $f_j << 1$, but not necessarily $\rho_c << 1$). Because of this fact, the same-order term $\mathbf{F}^{(s)}$ encompasses two (or more) nonlinear interactions that are usually regarded as different orders in conventional nonlinear optics where the polarization of the medium is expressed in powers of the applied field.[5] Such an "order mixing" is attributable to the finite size r_c of the cyclotron orbit.[6] However, the number of the orders s contributing to any particular nonlinear interaction is always limited and easily found. The force $\mathbf{F}_c^{(1)}$ is a nonzero quantity only for either the main resonance ($\omega = \Omega$) or for the generation of nth-order subharmonics ($\omega = n\Omega$); the latter one is a particular case of a multiphoton process. The subharmonic generation will be discussed elsewhere. The force $\mathbf{F}_c^{(2)}$, Eq. (7), contributes to three-photon (e.g., $\omega_1 - \omega_2 = \Omega$) and four-photon (e.g., $\omega_1 - \omega_2 = 2\Omega$) resonances both of which are considered below.

Three-photon resonance ($\omega_1 - \omega_2 = \Omega$).—Consider two optical waves (with their frequencies $\omega_1 > \omega_2 > 0$) counterpropagating in the direction *orthogonal* to the magnetic field $\mathbf{H}_0$ and linearly polarized with their vectors $\mathbf{E}_j$ *parallel* to $\mathbf{H}_0$ (see inset 1 in Fig. 1). In such a

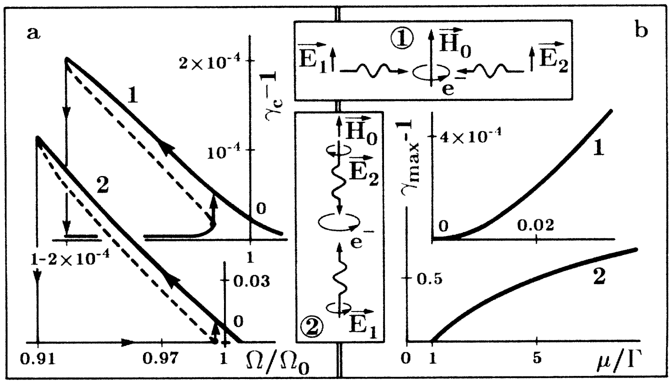

FIG. 1. (a) Dimensionless kinetic energy $\gamma_c - 1$ of the cyclotron electron motion vs the effective driving frequency Ω and (b) maximum kinetic energy $\gamma_{max} - 1$ vs driving parameter μ. Insets depict configurations of the two driving laser fields $\mathbf{E}_1$ and $\mathbf{E}_2$, with their propagations and polarizations shown with respect to the constant magnetic field $\mathbf{H}_0$, and the precession of the electron in the cases of (1) three-photon resonance, $\omega_1 - \omega_2 = \Omega$, and (2) four-photon resonance, $\omega_2 - \omega_1 = 2\Omega$. Curves 1, three-photon resonance, with $\mu_{(3)}/\Gamma = 0.02$; curves 2, four-photon resonance, with $\mu_{(4)}/\Gamma = 1.464$. The solid branches at curves in (a) correspond to stable states, and the broken branches to unstable states; the spacing between the solid and broken branches is grossly exaggerated in order to display the hysteresis.

case, in Eq. (4), $\mathbf{F}_c^{(1)} = 0$. In order to find the pumping threshold (which is very low in this case) required to observe relativistic features in the resonance, it is sufficient to consider the weak relativistic case when $\mathbf{r}_c$ in $\mathbf{f}_j(\omega_j t - \mathbf{k}_j \cdot \mathbf{r}_c)$ is small (i.e., $r_c << k_j^{-1}$ which amounts to $\rho_c << \Omega_0/\omega_j$). Assuming, e.g., $\mathbf{q}_1 = -\mathbf{q}_2 = \hat{\mathbf{e}}_x$, $\mathbf{h} = \hat{\mathbf{e}}_z$, and $\mathbf{f}_1 = f_1 \hat{\mathbf{e}}_z \sin(\omega_1 t)$, $\mathbf{f}_2 = f_2 \hat{\mathbf{e}}_z \sin(\omega_2 t)$, we find

$$\boldsymbol{\rho}_{\text{nc}}^{(1)} \simeq -\hat{\mathbf{e}}_z \Omega_0 \left[\omega_1^{-1} f_1 \cos(\omega_1 t) + \omega_2^{-1} f_2 \cos(\omega_2 t)\right].$$

Substituting this expression in Eq. (7) and solving Eq. (3) for a steady-state cyclotron motion $\boldsymbol{\rho}_c$, we obtain an equation for the magnitude ρ_c:

$$\rho_c = \mu_{(3)} \{\Gamma^2 + [(\Omega - \Omega_0)/\Omega + \rho_c^2/2]^2\}^{-1/2}, \quad (8)$$

where

$$\mu_{(3)} = f_1 f_2 \Omega_0 (\omega_1^{-1} + \omega_2^{-1})/4. \quad (9)$$

In this form, Eq. (8) is an exact analog of the equation[1,3] for the *main* resonance when ω is near Ω_0, the only difference being that instead of a (dimensionless) amplitude μ of the resonant driving wave, we now have a three-photon driving parameter, $\mu_{(3)}$. The cyclotron motion $\rho_c > 0$ may be excited with *any* magnitude of $\mu_{(3)}$. However, in order to obtain a hysteresis, which is a distinct signature of a relativistic resonance, one has to have $\mu_{(3)}$ exceed a threshold[1] $\mu_{cr} \simeq 1.75\Gamma^{3/2}$. Consider the case where $\Omega_0 = 2\pi \times 150$ GHz, i.e., $\lambda_0 = 2$ mm (and $\Gamma \simeq 0.6 \times 10^{-11}$), $\lambda_j (= 2\pi/k_j) \simeq 0.69$ μm (He-Ne laser), and $f_1 = f_2$. Making use of Eq. (9) we obtain a critical amplitude, $E_{cr} \simeq 6$ V/cm, which corresponds to an intensity of 48 mW/cm^2. If the beam is focused to a spot of ~ 2 μm diameter, this amounts to a total power of only 1.5 μW.

When the driving parameter $\mu_{(3)}$ considerably exceeds the threshold μ_{cr},the kinetic energy of the cyclotron motion, $\rho_c^2/2$, follows almost exactly the resonant detuning, i.e., $\rho_c^2/2 \simeq -(\Omega - \Omega_0)/\Omega_0$ (for $\Omega < \Omega_0$) [see curve 1 in Fig. 1(a)], until it reaches its maximum magnitude, $\rho_{c\,max}^2/2 = (\mu_{(3)}/\Gamma)^2/2$, which occurs when $(\Omega - \Omega_0)/\Omega_0 = (\mu_{(3)}/\Gamma)/\sqrt{2}$. Immediately after that, if $|\Omega - \Omega_0|$ continues to increase, the electron jumps from the higher excitation branch down to almost zero excitation, curve 1 in Fig. 1(a). A perturbation analysis of Eq. (3) in the vicinity of steady states reveals that the third branch of the steady-state solution, Eq. (8), which is located between the higher and lower branches (both of which are stable), is unstable. The maximum kinetic energy of excitation can be considerably high even for relatively low pumping. For instance, in order to obtain 1 MeV of excitation, the required power of a CO_2 laser ($\lambda_{1,2} \sim 10$ μm) with a focal spot of diameter $\lambda_0\gamma/\pi \sim 2$ mm is ~ 100 W.

Four-photon resonance $(\omega_1 - \omega_2 = 2\Omega)$.—Consider two optical waves counterpropagating along the z axis *parallel* to $\mathbf{H}_0$; both are circularly polarized now, in such a way that the polarization of the *higher*-frequency (ω_1) wave precesses around $\mathbf{H}_0$ in the *same direction as the electron motion,* whereas the *lower*-frequency (ω_2) wave polarization precesses in the *opposite* direction (see inset 2 in Fig. 1). In this case the fields are written as

$$\mathbf{f}_j = f_j \{\hat{\mathbf{e}}_x \sin[\omega_j t + (-1)^j k_j z] - (-1)^j \hat{\mathbf{e}}_y \cos[\omega_j t + (-1)^j k_j z]\}, \quad j = 1, 2.$$

In this case it is possible to make an analytic calculation for arbitrary cyclotron energy. Here again, $F_c^{(1)} = 0$. Assume $\boldsymbol{\rho}_c$ in the form

$$\boldsymbol{\rho}_c = \rho_c [\hat{\mathbf{e}} \sin(\Omega t + \phi) + \hat{\mathbf{e}}_y \cos(\Omega t + \phi)],$$

where ρ_c and ϕ are the slowly varying cyclotron momentum amplitude and phase, respectively. Equation (5) then yields

$$\boldsymbol{\rho}_{\text{nc}}^{(1)} \simeq (\Omega_0/\bar{\omega}) \{[\hat{\mathbf{e}}_z \times (\mathbf{f}_1 - \mathbf{f}_2)] + \hat{\mathbf{e}}_z \rho_c \gamma_c^{-1} [f_1 \sin(\bar{\omega} t - \phi) - f_2 \sin(\bar{\omega} t + \phi)]\},$$

where $\bar{\omega} = (\omega_1 + \omega_2)/2$. Substituting this into Eq. (7), solving Eq. (3) [with a force $\mathbf{F}^{(2)}$ given by Eq. (7)] for a steady-state cyclotron momentum, ρ_c [or total energy $\gamma_c = (1 + \rho_c^2)^{1/2}$], and introducing a four-photon driving parameter $\mu_{(4)}$,

$$\mu_{(4)} = 2(\Omega_0/\bar{\omega})^2 f_1 f_2 = 2e^2 E_1 E_2 (c/\bar{\omega})^2 (m_0 c^2)^{-2}, \tag{10}$$

one arrives at the results given below. In this four-photon process as distinct from the three-photon resonance, no cyclotron oscillation can be excited unless the driving parameter $\mu_{(4)}$ exceeds the threshold $\mu_{cr} = \Gamma$. Above this threshold the excitation becomes hysteretic and very large; the solution for energy becomes three-valued. One of these solutions is the nonexcited state, $\gamma_c = 1$, whereas the two other solutions are given by

$$\gamma_c = \frac{\Omega_0}{\Omega} + \left(\frac{\Omega}{\Omega_0}\right)^3 \left\{ -\mu_{(4)} \frac{(f_1^2 + f_2^2)}{4 f_1 f_2} \pm \left[\mu_{(4)}^2 - \Gamma^2 \left(\frac{\Omega_0}{\Omega}\right)^8 \right]^{1/2} \right\}, \tag{11}$$

except for a small gap near Ω_0 where only one of these solutions (the plus sign) exists. For each of the branches [upper, u, with the plus sign in Eq. (11), and lower, l, with the minus sign] with $\gamma_c > 1$, there are two solutions for the phase ϕ, given by

$$\phi_{u_{1,2}} = -\tfrac{1}{2} \arcsin[(\Gamma/\mu_{(4)})(\Omega_0/\Omega)^4] \pm \pi/2;$$

$$\phi_{l_{1,2}} = -\phi_{u_{1,2}} + \pi/2.$$

As the frequency Ω decreases, the total energy γ_c increases until it reaches the maximum magnitude $\gamma_{max} = (\mu_{(4)}/\Gamma)^{1/4}$ [see curves 2 in Figs. 1(a) and 1(b)] which occurs when $\Omega = \Omega_{cr} = \Omega_0 \gamma_{max}^{-1}$; immediately after that, the electron jumps to the nonexcited state. A perturbation analysis of Eq. (3) shows that again the upper branch of the solution, Eq. (11) (and phases ϕ_u), is stable, whereas the lower branch (and phases ϕ_l) is unstable. The nonexcited solution, $\gamma_c = 1$, is stable everywhere but at a small gap $\pm \Omega_0 \times (\mu_{(4)}^2 - \Gamma^2)^{1/2}$ around $\Omega = \Omega_0[1 - \mu_{(4)}(f_1^2 + f_2^2)/4 f_1 f_2]$. This gap gives rise to the inverse small jump from the nonexcited state to the stable excited state, the latter state given by Eq. (11) with the plus sign. The existence of two possible stable phases $\phi_{u_{1,2}}$ allows the electron to retain information concerning the initial conditions of excitation. Such a phase memory suggests an interesting model for a new, *phase* type of optical bistability.

We now estimate the critical intensity of laser pumping required to achieve four-photon cyclotron excitation. Assuming $\lambda_0 = 2$ mm, a CO_2 laser ($\lambda_{1,2} \approx 10$ μm), and $E_1 = E_2 = E$, and making use of Eq. (10) and the critical condition, $\mu_{cr} = \Gamma$, one obtains $E_{cr} = 0.5 \times 10^4$ V/cm which corresponds to $\sim 0.66 \times 10^5$ W/cm^2. With a focal spot size of ~ 45 μm in diameter, this corresponds to only 1 W of laser power.

A simple quantum consideration shows that the excited electron absorbs photons with the higher laser frequency, ω_1, and radiates at the lower laser frequency, ω_2 (which is analogous to stimulated Raman scattering) as well as at the cyclotron frequency Ω and its higher harmonics, which corresponds to stimulated emission at these frequencies.

It is known that the conduction electons in narrow-gap semiconductors demonstrate pseudorelativistic behavior[7] based on the nonparabolicity of the potential well and governed by the effective mass-energy relation similar to that for free electons (for the most recent experimental verification of this effect in InSb, see Zawadzki, Klahn, and Merkt[8]). Since such an effect may result in hysteretic cyclotron resonance in semiconductors,[9] the multiphoton hysteretic resonance in semiconductors, analogous to that of the case of free electrons in vacuum considered here, may also be expected, in particular in a cyclotron Raman laser[10] in semiconductors.

In conclusion, it has been demonstrated that the relativistic interaction of light and a single electron may result in multiphoton excitation of cyclotron motion by laser radiation with frequencies much higher than the cyclotron frequency. An excited cyclotron motion always exhibits a distinct relativistic hysteresis. The power of laser radiation required to observe these effects is surprisingly low and can be obtained in the cw regime of, e.g., He-Ne or CO_2 lasers.

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