# X-ray transition radiation in a solid-state superlattice: photoabsorption, electron scattering, and radiation optimization

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Soft x rays are generated when low-energy electron beams traverse a solid-state superlattice. We investigate the influence of x-ray absorption and electron scattering losses on the maximum power radiated, the required electron energy, and the optimum total thickness of the superlattice. We show that a moderate increase in the electron-beam energy compensates for the losses due to photoabsorption and electron scattering.

Transition radiation is generated when fast electrons traverse an interface between two media with different dielectric constants $\epsilon_1$ and $\epsilon_2$.[1] In a multilayer structure with many interfaces, the radiation is greatly enhanced and concentrates around some resonant angle $\theta_r$ measured from the trajectory of electrons (see the inset of Fig. 1). Each spatial mode $r$ distributes with a particular $\theta_r$ for a certain wavelength $\lambda$ and speed of electrons $v$:

$$\overline{\epsilon^{1/2}} \cos \theta_r = c/v - r\lambda/l, \tag{1}$$

where $\overline{\epsilon^{1/2}} = (\sqrt{\epsilon_1}l_1 + \sqrt{\epsilon_2}l_2)/l$ is the average refractive index, $l_1$ and $l_2$ are the thicknesses of the individual layers formed by two media, and $l = l_1 + l_2$ is the spatial period of the periodic medium. Constructive interference results in the resonant condition of Eq. (1), which was recently confirmed by experiment[2] for a limited number of layers. Previously,[1-3] periodic structures with $l \gg \lambda$ were suggested, thus requiring ultrarelativistic electrons with energies of 100 MeV–50 GeV,[3] to satisfy condition (1) for real $\theta_r$. Recently developed technologies are capable of growing multilayer structures with layers less than 10 nm thick. These structures are widely used as soft-x-ray mirrors.[4] The generation of soft x rays by using a low-energy electron beam that passes through such a multilayer structure was recently proposed.[5,6] In solid-state structures, however, electron scattering and photoabsorption may be the main obstacles to achieving an effective source of x-ray radiation.

In this Letter we study the losses due to photoabsorption and electron scattering and their effect on resonant transition radiation. We obtain the optimum parameters for the multilayer medium and the maximum radiation power for a given frequency $\omega$. We show that above an electron energy of ≈100–200 keV for most materials and frequencies, the effect of photoabsorption becomes dominant over electron scattering. We also show that photoabsorption imposes an energy ceiling above which an increase in electron-beam energy produces no significant improvement.

The differential cross section for transition radiation in a multilayer medium can be expressed as follows[1,3]:

$$\frac{d^2N}{d\Omega d\omega} = F_1F_2F_3, \tag{2}$$

where $N$ is the number of photons per electron, $\omega = 2\pi c/\lambda$, $\Omega$ is the solid angle, $F_1$ is the differential cross section for a single interface, $F_2$ denotes the coherent interference of the radiation in a single plate (i.e., two neighboring interfaces), and $F_3$ represents the coherence summation of radiation from each layer. The factor $F_1$ given by[1]

$$F_1 = \alpha(\Delta\epsilon\beta)^2|G(\beta, \theta)|^2/(\pi^2\omega), \tag{3}$$

where $\Delta\epsilon = \epsilon_1 - \epsilon_2$, $\alpha \approx 1/137$ is the fine-structure constant, $\theta$ is the angle of photon emission, $\omega$ is the

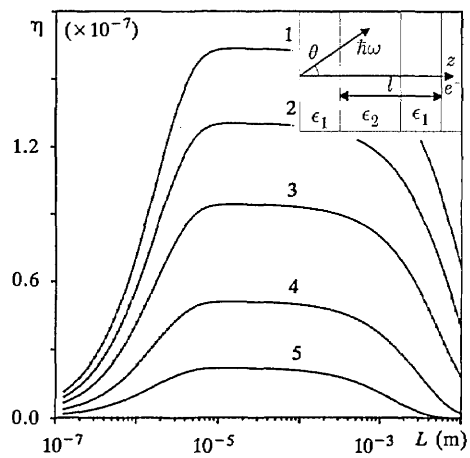


Fig. 1. Transition radiation efficiency $\gamma$ versus total length of the structure $L$ (in meters) at a wavelength ~1.7 nm for the Be/Ba combination at different electron-beam energies $E_0$. Curve 1, $E_0$ = 4 MeV; curve 2, 3 MeV; curve 3, 2 MeV; curve 4, 1 MeV; and curve 5, 0.5 MeV. Inset: The configuration of the x-ray source.

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frequency of the radiation, $\beta = v/c$, and $G(\beta, \theta)$ is the radiation pattern of a single interface. If the electron energy is lower than that required for Čerenkov radiation (i.e., less than 100–500 MeV in our case), $G$ can be expressed as

$$G(\beta, \theta) = \sin\theta(1 - \beta^2 - \beta\cos\theta)/[2 - (1\beta^2\cos^2\theta)(1 - \beta\cos\theta)]. \quad (4)$$

The factor $F_2$ is given by $F_2 = 4\sin^2[l_2\pi(1/\beta - \sqrt{\epsilon_2}\cos\theta)/\lambda]$; its maximum, $F_2 = 4$, is achieved, e.g., when $l_1 = l_2 = l/2$.

Photoabsorption and electron scattering affect only the multilayer factor, $F_3$. To include photoabsorption, we assume that the electric field decays exponentially as $\exp[-\bar{\mu}z/(2\cos\theta)]$, where $\bar{\mu} = (\mu_1 l_1 + \mu_2 l_2)/l$ is the average absorption coefficient and $z$ is the distance traveled by the electron. For $M$ periods we have

$$F_3 = \left|\sum_{s=0}^{M-1}\Phi_s\right|^2, \qquad \Phi_s = \exp[-\sigma(M - s) + 2iXs], \quad (5)$$

where $2X = 2\pi l(1/\beta - \cos\theta\epsilon^{1/2})$ is the phase retardation for a single period of the structure and $\sigma = \bar{\mu}l/(2\cos\theta)$ is a dimensionless absorption parameter. Although both the energy and the momentum of the electrons change as the electrons pass through a layer of the material, most of the transmitted electrons conserve their energy and momentum if the film is thinner than the mean free path.[7] The number of these surviving electrons can be described by the transmission $T$. In the first approximation, we consider only the radiation contribution of these electrons. Interpreting $T(z)$ as the probability that an electron passes through a distance $z$, we can account for electron scattering by redefining $\Phi_s$ in Eq. (5) as $\Phi_s = \sqrt{T(sl)}\exp[-\sigma(M - s) + 2iXs]$. (Previously we used a different, less rigorous, approach.[8] The results of Ref. 8 depart from the present calculations by less than an order of magnitude.) The dependence of $T(z)$ on the thickness of the medium, $z = sl$, can be characterized by the so-called critical length $L_{cr} = (dE/dz)$, where $dE/dz$ is the energy loss of the electrons per unit distance and $E_0$ is the initial electron energy. $L_{cr}$ is defined as the length for which virtually all the electrons are scattered and absorbed, and it can be calculated by using Bethe's formula.[9] For $L_{cr}$ longer than the total length of the multilayer structure $Ml$ (which is usually true), $T$ decays almost exponentially, $T = \exp(-z/L_{cr})$. It can be shown then that the factor $F_3$ is expressed as

$$F_3 = \exp[-(M - 1)(\gamma + \rho)]\{\cosh[M(\sigma - \rho)] - \cos(2MX)\}/[\cosh(\sigma - \rho) - \cos(2X)], \quad (6)$$

where $\rho = l/(2L_{cr})$. Because of the rapid variation of $F_3$ with angle $\theta$, it is more meaningful to measure the radiation yield by integrating Eq. (2) over solid angle $\Omega$, using Eqs. (3) and (6) under the condition that $r = 1$ and $l_1 = l_2 = l/2$:

$$\frac{dN}{d\omega} = \int_{-\infty}^{\infty}\left(\frac{d^2N}{d\omega d\Omega}\right)d\Omega = \frac{8\beta^2\lambda^2\alpha}{\pi^2 cl}\frac{\sinh[M(\sigma - \rho)]}{(\sigma - \rho)} \times \exp[-M(\sigma + \rho)]|\Delta\epsilon|^2|G(\beta, \theta)|^2. \quad (7)$$

Equations (6) and (7) reduce to the known respective formulas[3] when only photoabsorption is present (i.e., $\rho = 0$). Bremsstrahlung radiation is the chief process competing with transition radiation. However, in a typical case for a proposed source (e.g., $L \sim 10$ $\mu$m, $\lambda \sim 1.6$ nm, $E_0 \sim 4.5$ MeV, and $Be/Ba$ as the medium), transition radiation can be shown to be 1 to 2 orders of magnitude stronger than bremsstrahlung radiation.

To optimize the radiated power at a desired frequency, we have to minimize losses and, more importantly, maximize the magnitude of $\Delta\epsilon$ around the desired frequency. Because of resonant anomalous dispersion, the dielectric constant undergoes drastic changes[10] near the frequencies of the atomic absorption edges; this is attributed primarily to photoionization from the inner atomic shells. At the frequency just below an absorption edge, photoabsorption is relatively weak, while the dielectric constant can drastically increase. Therefore large $\Delta\epsilon$ and low photoabsorption $\bar{\mu}$ can be achieved by choosing one of the constitutive materials to have an atomic absorption edge near a desired radiation frequency while requiring that the other material be nonresonant in the same range.[11] We now use Eq. (7) to develop a procedure for selecting geometrical structure factors so that the radiation is maximized in the desired frequency range. We maximize the right-hand side of Eq. (7) with respect to $\theta$ and hence obtain $\theta_r$ and the optimal spatial period $l_{opt}$. We then maximize the term $\sinh[(\sigma - \pi)M]\exp[-M(\sigma + \rho)]/(\sigma - \rho)$ with respect to $M$ and obtain the optimum number of periods as $M_{opt} = [\ln(\sigma/\rho)]/[2(\sigma - \rho)]$.

The dependence of radiation efficiency, defined here as $\eta = (dN/d\omega)E_0^{-1}$, on total length $L = Ml$ is depicted in Fig. 1. After reaching a maximum, the radiation efficiency $\eta$ plateaus at a length $L \approx 1/\bar{\mu}$, which can be explained by the domination of photoabsorption for short lengths $L < L_{cr}$ and by the domination of electron scattering at longer lengths $L \gg L_{cr}$. We can now determine the maximum radiation for angle $\theta_r$, period $l_{opt}$, and number of periods $M_{opt}$. An optimized Eq. (7) can now be written as

$$\frac{dN}{d\omega} = \frac{\alpha}{2\pi^2 c}|\Delta\epsilon|^2\frac{1}{\bar{\mu}}PQ, \quad (8)$$

where

$$Q = (1 - \beta l/\lambda)^2[1 - (1/\beta - \lambda/l)^2](1/\beta - \lambda l),$$
$$P = (\rho/\sigma)^{\rho/(\sigma-\rho)}. \quad (9)$$

One can see that for sufficiently high electron-beam energy (e.g., $E_0 > 1$ MeV), $Q_{opt}$ and $l_{opt}$ are approximated by a simple formula:

$$Q_{opt} \approx l_{opt}/\lambda \approx \gamma^2. \quad (10)$$

For a fixed $l$, $Q$ saturates[3] after $Q_{opt}$ is reached, even if the electron energy keeps increasing. We discovered,

**Table 1. Optimal Spatial Period $l_{opt}$, the Optimal Total Length $L_{opt}$, and the Transition Radiation Efficiency $\eta$ for the Be/Ge, Be/Ce, Be/Ba, and Be/Eu Superlattice Structures[a]**

| Radiator | $\lambda$ (nm) | $l_{opt}$ (nm) | $L_{opt}$ ($\mu$m) | $\eta$ |
|---|---|---|---|---|
| Ge | 1 (1.20 keV) | 9.4 | 18.1 | $4.09 \times 10^{-6}$ |
| Ce | 1.4 (0.88 keV) | 13 | 8.3 | $2.77 \times 10^{-6}$ |
| Ba | 1.6 (0.77 keV) | 14.6 | 12.8 | $2.89 \times 10^{-6}$ |
| Eu | 1.1 (1.11 keV) | 10.1 | 13.1 | $1.08 \times 10^{-7}$ |

[a] These values are for an electron-beam energy $E_0$ = 1 MeV and are at radiation peaks for transitions in the heavier elements (the $L$ edge for Ge and the $M$ edge for Ba, Ce, and Eu). The angle of emission is $\theta_r = 17.5°$.

however, that even when the period $l$ is optimized for each $E_0 = (\gamma - 1)m_0c^2$, there is some *maximum* meaningful energy (ceiling) of the electron beam that is related to the photoabsorption. Indeed, it is obvious that the use of a multilayer structure thicker than $1/\bar{\mu}$ will not yield a further increase in radiation. On the other hand, in order to have a multilayer structure with a reasonable number of layers (say, $M \geq 4$),[2] the condition for the maximum spatial period of the structure, $l \leq 1/4\bar{\mu}$, must be taken into account. Using this consideration as well as expression (10) for $l = l_{opt}$, we obtain the ceiling on the required $\gamma \leq 1/(2\sqrt{\lambda\bar{\mu}})$. With a typical value of $1/\bar{\mu}$m at $\lambda \approx 1$ nm, we obtain $\gamma \approx 15$, corresponding to $E_0 \approx 7$ MeV. For light materials this ceiling may reduce to 2 MeV or even lower.

The *minimum* electron-beam energy required to obtain appreciable radiation intensity is achieved when the magnitude of the electron scattering is equal to the magnitude of the photon absorption, i.e., $L_{cr}/e = 1/\bar{\mu}$. The typical value of $1/\bar{\mu}$ is of the order of micrometers for a wavelength of 1 nm. For example, when $1/\bar{\mu} \approx 5$ $\mu$m, the minimum electron energy is approximately 100 keV. Above this energy, photoabsorption begins to dominate over electron scattering.

We consider various combinations of different media that constitute the periodic structure. In these examples, our choice of a light element (spacer) is beryllium (Be) since it has low absorption loss in the soft-x-ray frequency range. For the sake of illustration, we choose one of the elements barium (Ba), cerium (Ce), europium (Eu), or germanium (Ge) as a heavy element (radiator). In the general case, the selection of a heavy element is determined mainly by one of its atomic edges, which has to coincide with a desired radiation frequency.

The optimized parameters $L_{opt}$ and $l_{opt}$ at some resonant frequency for the Be/Ba, Be/Ce, Be/Ge, and Be/Eu structures are listed in Table 1 along with the corresponding radiation efficiency $\eta$. The value of parameters $L_{opt}$ and $l_{opt}$ in Table 1 correspond to structures that are optimized at the $M$ absorption edges of the heavy elements (except Ge, for which we choose the $L$ edge). To compare our scheme with other methods[3] that use high-energy electrons (e.g., 100 MeV–50 GeV), we define the radiation efficiency as $\eta = (dN/d\omega)E_0^{-1}$, which differs from the conventional definition by the normalization with respect to electron-beam energy. This normalization addresses the cost of using a high-energy beam. Typically, the radiation efficiency of the proposed method (up to $\eta \approx 10^{-7}$ at $E_0$ = 1 MeV) is about the same order of magnitude as the radiation efficiency of the previously known method (e.g., $\eta \approx 10^{-7}$ in Ref. 3).

In conclusion, we have demonstrated the role of photon absorption and electron scattering in x-ray transition radiation in multilayer solid-state structures using electrons with relatively low energies. We have used these results to develop an optimization procedure whereby we find the optimum parameters of the structure required to obtain maximum radiation. We have shown that an electron-beam energy $E_0$ in the range from 100 or 200 keV to a few megaelectron volts is most appropriate to produce resonant x-ray transition radiation in these structures.

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