Dispersion-related multimode instabilities and self-sustained oscillations in nonlinear counterpropagating waves

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We show that two linearly polarized counterpropagating waves in a Kerr nonlinear medium with linear dispersion can exhibit multimode temporal instability. We find the boundary of the unstable regime and demonstrate that the fully developed instability results in self-sustained oscillations and the onset of chaos.

The cross interaction of two counterpropagating light beams[1-6] in a third-order nonlinear material is a perceptionally simple but fundamental process with a host of potential applications. The steady states of this interaction have been shown to exhibit hysteresis and optical bistability,[1] nonlinear eigenpolarizations,[2] multivalued solutions with hysteretic, isolated (nonhysteretic) solutions,[3] and the possibility of chaotic *spatial* behavior.[4] On the other hand, the *temporal* stability of intense counterpropagating beams is of great importance for such nonlinear systems as lasers and optical bistable devices. A nonlinear medium having a finite relaxation time has been shown to exhibit temporal instability and chaos as the driving intensity exceeds a certain threshold[5] for the simplest eigenpolarization (with two counterpropagating beams having linear polarizations parallel to each other). Subsequently periodic and chaotic temporal behavior was demonstrated in the same system for various nonlinear eigenpolarizations.[6] However, the relaxation of the nonlinear refractive index may not be the most likely mechanism of instability since it imposes too stringent requirements on the relaxation time.

In this Letter we show that another factor, namely, regular linear dispersion (i.e., frequency dependence of the refractive index), can be a natural and universal agent for temporal instability in the system. Although linear dispersion can give rise to well-known nonlinear optical effects such as spatial instability of a single plane wave,[7] solitons in nonlinear fibers,[8] and amplification of copropagating waves,[9] this factor has never been discussed, to our knowledge, in application to the problem in consideration.[10]

We assume an absence of losses and investigate the simplest spatially stable eigenpolarization[2,3] whereby the two waves are linearly polarized with their electrical fields parallel to each other. (Note that spatially unstable eigenpolarizations may result in temporal instability even if no other mechanism is present.[11]) The complex electric field is then represented as $\mathbf{E} = [E_1(z, t)\exp(ikz) + E_2(z, t)\exp(-ikz)]\exp(-i\omega t)\hat{e}_x$, where $E_1(z, t)$ and $E_2(z, t)$ are envelopes of forward ($+z$) and backward ($-z$) propagating waves, respectively, whose behavior in the third-order nonlinear medium with linear dispersion is governed by

$$i\left[(-1)^{j+1}\frac{\partial E_j}{\partial z} + \frac{1}{v_g}\frac{\partial E_j}{\partial t}\right] - \frac{\mu}{2}\frac{\partial^2 E_j}{\partial t^2} = -\beta(2I_{3-j} + I_j)E_j, \qquad j = 1, 2, \quad (1)$$

where $I_j = |E_j|^2$ is the intensity of the respective propagating wave, $\mu = \partial^2 k/\partial\omega^2$ is the linear dispersion parameter, v_g is the linear group velocity, $\beta = \chi k/2$ is the nonlinear parameter, χ is a constant of nonlinear interaction, and k is the wave number in the medium. The coefficient 2 on the right-hand side of Eq. (1) reflects light-induced nonreciprocity.[12] The problem of temporal instability of cw wave propagation for certain signs of dispersion and nonlinearity could be isomorphic to the problem of spatial instability of plane counterpropagating waves in a nonlinear medium[13] (which can ultimately result in cross-induced self-focusing bistability[14]), since the latter problem can be described by Eq. (1), in which the term $\partial^2E/\partial t^2$ is replaced by $\partial^2E/\partial x^2$, where x is a coordinate in the transverse plane (i.e., spatial dispersion). The steady-state solution for Eq. (1) is $I_j =$ const. $= I_{j0}$ and $E_{js}(z) = (I_{j0})^{1/2}\exp\{i\beta[I_{j0} + 2I_{(3-j)0}][z - (j - 1)L]\}$, where L is the length of the nonlinear medium. There is *no* exchange of energy between the two beams in the *steady state*.[2,3] To analyze the linear stability of the steady state, we assume a perturbed solution of Eq. (1) in the form

$$E_j = E_{js}(z)[1 + \delta A_j(z)\exp(\lambda t) + \delta B_j^*(z)\exp(\lambda^* t)], \qquad j = 1, 2, \quad (2)$$

where $\delta \ll 1$, A_j and B_j are some (unknown) functions of z, and λ is some (unknown) complex constant (eigenvalues). The perturbed solution is unstable if $\mathrm{Re}(\lambda) > 0$. At this point we simplify our calculation, assuming equal intensities of the waves, i.e., $I_{10} = I_{20} = I$. Using the boundary conditions for A_j and B_j, $A_1(z = 0) = A_2(z = L) = B_1(z = 0) = B_2(z = L) = 0$, one can obtain an equation for normalized complex eigenvalue λ. The boundaries of unstable regions are found by setting $\mathrm{Re}(\lambda) = 0$ and solving for the normalized driving intensity $p = \beta IL$ (note that p can have negative sign depending on nonlinearity β) and $\mathrm{Im}(\lambda)$ for a given dispersion.

The numerical results of this procedure are depicted

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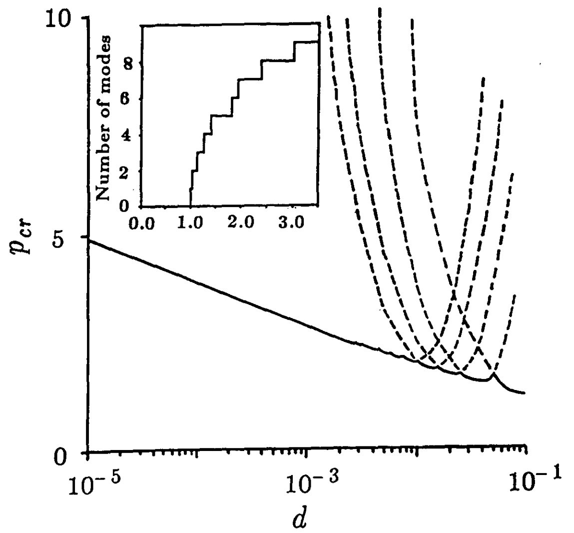

Fig. 1. Normalized threshold intensity p_{cr} versus the normalized dispersion d. The solid curve is the boundary of instability above which the system is unstable. The dashed curves indicate the boundaries of the instability of individual modes. The inset shows the number of unstable modes versus $|p|/p_{cr}$, where p is the normalized driving intensity.

in Fig. 1, which shows the normalized threshold intensity $p_{cr} = |\beta| I_{cr} L$ (I_{cr} is the threshold intensity) versus the normalized dispersion $d = \mu v_g^2/(2L)$ in a semilog plot. The solid (bounding) curve is the boundary above which the system is unstable. Below this curve the system is stable, although it can exhibit strong amplification at the frequencies adjacent to the pumping frequency. In the region above the threshold there are numerous solutions for the boundary of instability, with each one corresponding to an individual mode of oscillation that can be viewed as the longitudinal modes in a light-induced, distributed-feedback resonator. Each mode has its detuning frequency [Im(λ)] approximately $3v_g/L$ apart in frequency from the adjacent modes. Only a few of these solutions and the envelope encompassing all the unstable solutions (i.e., the solid curve) are shown in Fig. 1. In the inset of Fig. 1 the number of unstable modes is plotted against $|p|/p_{cr}$, where $p_{cr} = 1.98$ is the threshold intensity for the case of $d = 0.01$. The (bounding) threshold intensity increases as dispersion decreases; for sufficiently small d ($\ll 1$), this can be best described by a surprisingly simple relationship:

$$p_{cr} = -\mathrm{sgn}(\beta d)|\log_\eta |d||, \qquad (3)$$

where η = const. $\simeq 10$ and sgn(βd) is the sign of βd. The necessary condition for initiating instability is that the signs of nonlinearity and dispersion must be opposite, which coincides with the necessary condition for formation of a soliton and spatial instability in single-wave propagation.

To explore the full dynamical behavior in the unstable region shown in Fig. 1, we have numerically integrated Eq. (2) in which we ramp the input fields slowly in time to the desired steady-state input intensity and maintain them at this level with the dispersion fixed at $d = 0.01$. In the unstable region (i.e., $|p| > p_{cr} = 1.98$) we let the instability be excited by the computer noise. The slow increase of intensity with normalized time, $T = tv_g L^{-1}$, when the intensity is slightly greater than the threshold ($|p| = 2.02$), is shown in Fig. 2. At such a driving intensity, only one mode is unstable (see the inset in Fig. 1) and eventually develops into periodic self-sustained oscillations with a stable amplitude. When $|p|$ is further increased to 2.5 the instability and resulting oscillations develop much faster, as shown in Fig. 3. In this case *three* modes satisfy the excitation condition (see the inset in Fig. 1); as the oscillations develop in time the second subharmonic is excited, which later evolves into aperiodic oscillations with randomly modulated amplitude and phase, indicating the onset of chaos. This behavior further develops into strongly pronounced chaos when the driving intensity is significantly higher than the threshold, and therefore more unstable modes with different eigenfrequencies will survive and compete.

Using a 1-km-long single-mode fiber with a Ge-doped silica core and a wavelength of 1.55 μm, a group-velocity dispersion $D(k) = 6.5 \times 10^{-3}$ [note that $\mu = D(k)k^{-1}c^{-2}$], a linear refractive index of 1.44, and $\chi = 3.2 \times 10^{-16}$ cm^2/W,[15] in the lossless approximation we find that the threshold intensity I_{cr} [Eq. (3)] for such a fiber is 6.7×10^8 W/cm^2, which is below the damage threshold of 10^{10} W/cm^2 for fused silica.[16] A raw estimate shows that the losses existing in the real fiber [~0.5 dB/km (Ref. 15)] would require approximately a twofold higher threshold intensity. If we use SF-59 glass with $\chi = 7 \times 10^{-15}$ cm^2/W (Ref. 17) at a wavelength of 1.06 μm and assume that $D(k) = 12.7 \times 10^{-3}$, the same as for plain glass, the critical intensity is reduced to $\sim 2 \times 10^7$ W/cm^2 for the same length.

In conclusion, we have shown that linear frequency dispersion together with Kerr nonlinearity can result in temporal instability of nonlinear counterpropagat-

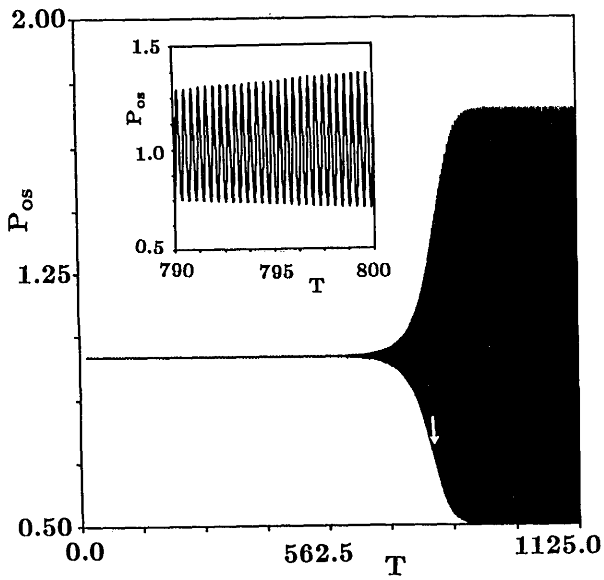

Fig. 2. Normalized intensity p_{os} of oscillations versus the normalized time T for a driving intensity $|p| = 2.02$ (above-threshold pumping, $|p|/p_{cr} = 1.02$, with single-mode excitation) when normalized dispersion d is fixed at 0.01. The inset is an enlargement of the region indicated by the arrow and demonstrates the single-mode nature of excitation. The frequency of oscillations is $\simeq 17.2 v_g L^{-1}$.

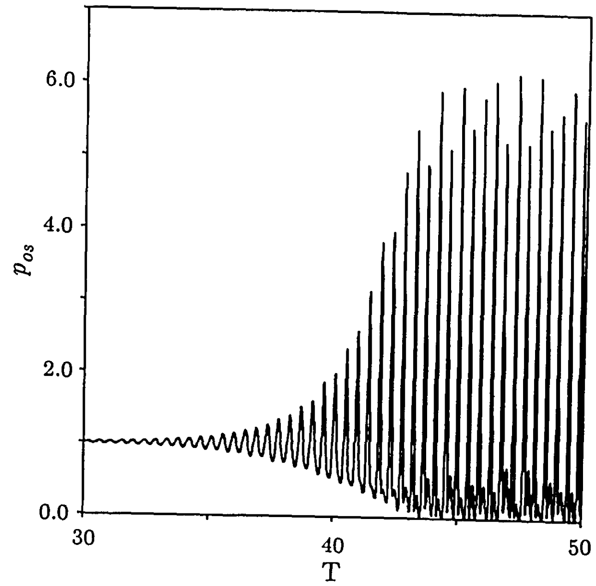

Fig. 3. The same conditions as in Fig. 2 except with a driving intensity $|p| = 2.5$ ($|p|/p_{cr} = 1.25$ with three unstable modes involved). The excitation shows the formation of a subharmonic and the onset of chaos. The initial frequency of oscillations is slightly less than that shown in Fig. 2.

ing waves if their intensities exceed a certain threshold. This suggests that regular linear dispersion can be a universal agent for amplification of perturbations and the resulting instability (and self-oscillations) in the nonlinear counterpropagating waves. The mechanism of these effects can be explained in terms of positive distributed feedback from the nonlinear index grating formed by two laser beams in the presence of dispersion. The feedback is attributed to the fact that small perturbations with different frequencies propagate with phase speeds that are different (owing to dispersion) from that of the fundamental wave; this gives rise to the energy exchange between the two beams at the nonlinear index grating (such an exchange is prohibited for a steady state or in a nondispersive medium).

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