Ding and Kaplan Reply: In the preceding Comments by Raizen and Rosenstein[1] (hereafter RR) and by Ford and Steel[2] (hereafter FS) on our recent Letter,[3] RR state that the QED box diagram for second-harmonic generation (SHG) in a dc magnetic field vanishes identically, whereas using phenomenological results, FS state that although nonlinearity of the respective order does not vanish, SHG vanishes in the case of collinear propagation.

We agree with RR that the result for the box diagram vanishes only for *collinear* photons[4] and when dispersion is absent. The main reason for the box diagram vanishing is nonconservation of four-momentum in such an interaction.[4,5] However, in the case of weak dispersion in vacuum (which always exists intrinsically[5]), the condition of four-momentum conservation is less restrictive for SHG. This results in the fact that even in the box approximation, SHG does not vanish (although the box-diagram contribution is smaller than that from the hexagonal diagram); see the Appendix in Ref. 5 which points out that dispersion can enhance the SHG amplitude. Our preliminary calculations show that the effect proposed in Ref. 3 can still be observed if one spatially modulates the amplitude of the dc magnetic field, by using, e.g., wigglers, which changes the dispersion of the system. Regarding the statement in RR that "more complicated diagrams" must be used even in the case of noncollinear photons, we want to stress again that essentially the box approximation is still valid and nonvanishing in the case of slight noncollinearity (see the discussion below as well as FS's Comment that still uses results of the box approximation).

Regarding the comments by FS we want to point out that the sign in the nonlinear constitutive relations used by us was based on Eqs. (54.30) in Ref. 6. Other sources, in particular the original publications[7] and those cited in FS, seem to use the opposite sign for **M**, the difference being attributed to the fact that the notations for **H** and **B** have been interchanged in Ref. 6. Presuming the propagation equations in vacuum in the standard form of Maxwell's equations,[8]

$$\nabla\cdot\mathbf{B}=0,\quad \nabla\times\mathbf{E}+(1/c)\,\partial\mathbf{B}/\partial t=0\,, \tag{1}$$

$$\nabla\cdot\mathbf{D}=0,\quad \nabla\times\mathbf{H}-(1/c)\,\partial\mathbf{D}/\partial t=0\,, \tag{2}$$

with the nonlinear constitutive relations in the form in Eqs. (3) in FS, we agree with FS that if one chooses **E** and **B** as base vectors describing the wave propagation, and use nonlinear components of vectors **D** and **H** as driving terms for Maxwell's equations, then the driving terms for collinear propagation vanish as in FS. However, although **E** and **B** are fundamental vectors, the propagation of *energy* (and therefore energy flow in SHG) in classical electrodynamics seems to be based on the vectors **E** and **H** since only these vectors form the Poynting vector, $\mathbf{S}=(c/4\pi)(\mathbf{E}\times\mathbf{H})$,[9] not **E** and **B**. Using Eqs. (1) and (2), we obtain the equations for $\mathbf{E}_2$ and $\mathbf{H}_2$ at the second-harmonic frequency with nonvanishing driving terms; for a particular polarization configuration, in which the fundamental wave is polarized along the dc magnetic field, the driving terms are

$$\begin{aligned}\mathbf{D}^{(2)}&=\hat{\mathbf{e}}_x A\exp[2i(k_1y-\omega_1t)]\,,\\ \mathbf{B}^{(2)}&=\hat{\mathbf{e}}_z A\exp[2i(k_1y-\omega_1t)]\,,\end{aligned} \tag{3}$$

where $A=\frac{7}{2}\xi E_1^2H_0$, although these driving terms do not lead to the same results as in Ref. 3. The difference between results for the pairs **E**,**H** and **E**,**B** disappears for the noncollinear fundamental beams. Since the driving terms do not vanish for SHG in a dc magnetic field in the general case (as is also indicated in FS), they should give a nonvanishing result for noncollinear propagation, in which case the calculation[3] gives the estimate which we believe is correct to the order of magnitude.

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Y. J. Ding and A. E. Kaplan
Department of Electrical and Computer Engineering
The Johns Hopkins University
Baltimore, Maryland 21218

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