Dispersion-related amplification in a nonlinear fiber pumped by counterpropagating waves

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We investigate the dispersion-related amplification induced by two counterpropagating waves in a nonlinear fiber and demonstrate that this system can provide broadband (of the order of terahertz) and large ($>10^5$) amplification even in the presence of absorption, with the pumping significantly below the threshold of instability.

A simple interaction of two counterpropagating waves in a medium with a nonlinear refractive index can result in numerous effects.[1-6] These effects in the steady-state regime include hysteresis and optical bistability,[1] light-induced nonreciprocity,[2] nonlinear eigenpolarizations,[3,4] multivalued solutions with both hysteretic and isolated (nonhysteretic) solutions,[3] and possibility of chaotic spatial behavior.[4] Temporal dynamics of counterpropagating nonlinear waves[5-7] is even more fascinating and is of great significance to lasers, optical gyroscopes, and various nonlinear-optical bistable devices. Recently we showed[7] that regular linear dispersion in combination with nonlinear processes in such a system can be a natural and universal agent for amplification of perturbations and for temporal instability if the pumping is sufficiently strong.

In this Letter we find that when the pumping intensity is below the critical intensity of instability, the amplification can still be large ($>10^5$) and has a large bandwidth (of the order of terahertz) and that linear absorption in existing fibers only slightly influences the magnitude of amplification. Our results suggest that a typical optical fiber under moderate pumping can be used as an amplifier in the optical window near the wavelength of 1.55 μm and that, therefore, the proposed effect has potential for signal amplification in long-distance optical fiber networks.

We consider the simplest spatially stable eigenpolarization[2,3] whereby the two waves are linearly polarized with their electrical fields parallel to each other. The total complex electric field inside the third-order nonlinear medium is expressed as $\mathbf{E} = [E_1(z, t)\exp(ikz) + E_2(z, t)\exp(-ikz)]\exp(-i\omega t)\hat{e}_x$, where $E_1(z, t)$ and $E_2(z, t)$ are the slowly varying envelopes of forward ($+z$) and backward ($-z$) propagating waves. The dynamics of these envelopes is governed by two coupled nonlinear Schrödinger equations,[7]

$$i\left[(-1)^{j+1}\frac{\partial E_j}{\partial z} + \frac{1}{v_g}\frac{\partial E_j}{\partial t}\right] - \frac{\mu}{2}\frac{\partial^2 E_j}{\partial t^2} = -\beta(2I_{3-j} + I_j)E_j - i\alpha E_j/2, \qquad j = 1, 2, \quad (1)$$

where $I_j = |E_j|^2$ is the intensity of the respective propagating wave, $\mu = \partial^2 k/\partial\omega^2$ is the linear dispersion parameter, k is the wave number in the medium, $\beta = n_2 k/n$ is the nonlinear parameter, n is the linear refractive index at the frequency ω, n_2 is the nonlinear refractive-index coefficient, v_g is the linear group velocity, and α is the linear absorption coefficient. The coefficient 2 in the right-hand side of Eq. (1) reflects light-induced nonreciprocity.[2]

To investigate the dispersion-related amplification, we consider a pump–probe configuration.[7] It consists of three inputs: two strong counterpropagating pumping waves with incident wave amplitudes E_{10} and E_{20} with frequency ω, and a weak signal wave ($E_{10}A_{1,1}$) with a frequency of $\omega + \delta\omega$. In addition to the amplified (forward) output at the same normalized frequency $\omega + \delta\omega$, there are also three other weak outputs. They are generated as a result of cross-phase interaction: a backreflected phase-conjugate signal ($E_{20}A_{2,2}$) with a frequency $\omega - \delta\omega$, and cross-coupled forward ($E_{10}A_{2,1}$) and backward ($E_{20}A_{1,2}$) waves with frequencies of $\omega - \delta\omega$ and $\omega + \delta\omega$, respectively.[8]

To analyze small-signal gain, we represent total forward and backward propagating waves as combinations of strong pumping waves with small amplitude waves,

$$E_j = E_{j0}\exp(i\phi_j)\{\exp[(-1)^j\gamma(\xi + 1 - j)] + A_{1,j}(\xi)\exp(-i\delta\Omega\tau) + A_{2,j}^*(\xi)\exp(i\delta\Omega\tau)\}, \qquad j = 1, 2, \quad (2)$$

where $\phi_j = \beta\{I_{j0} + 2I_{(3-j)0}\exp[(-1)^j(2 - j - \xi)\gamma]\}\{1 - \exp[(-1)^j\gamma(\xi + 1 - j)]\}/\gamma$, $\gamma = \alpha L$ is the normalized absorption coefficient, $A_{1,j}$ and $A_{2,j}$ are normalized amplitudes ($|A_{k,j}| \ll 1$), $\tau = tv_g/L$ is the normalized time, $\delta\Omega = \delta\omega L/v_g$ is the normalized detuning frequency, and $\xi = z/L$ is the normalized distance of propagation. Substituting Eq. (2) into Eq. (1) and linearizing the latter equation with respect to small amplitudes $A_{k,j}$, we obtain four linear propagation equations for $A_{k,j}$:

$$\frac{dA_{k,j}}{d\xi} = i(-1)^{k+j}\{A_{k,j}[p_j - (-1)^k(\delta\Omega + i\gamma/2) + d\delta\Omega^2] + p_j A_{3-k,j} + 2p_{3-j}\exp[(-1)^j\gamma(\xi - 1/2)] \times (A_{1,3-j} + A_{2,3-j})\}, \qquad j, k = 1, 2, \quad (3)$$

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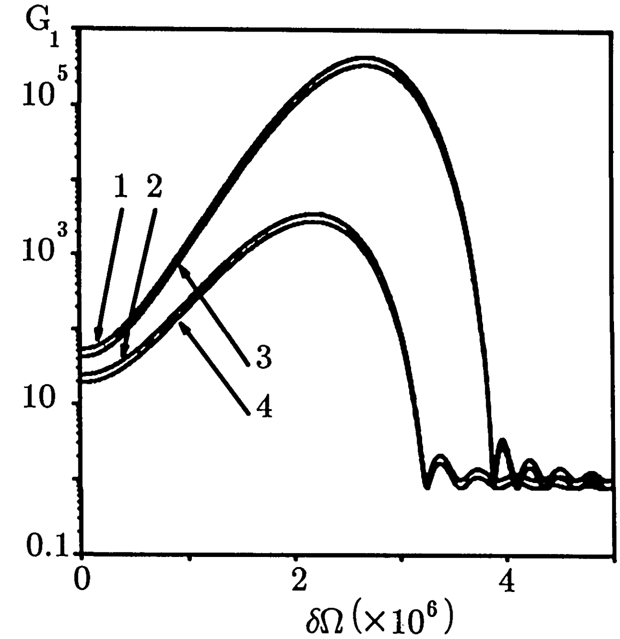

Fig. 1. Amplification G_1 of the signal wave versus the normalized frequency detuning $\delta\Omega$ for a 1-km-long Ge-doped silica fiber (see details in the text) for different normalized absorption coefficients γ (or α). Curve 1, $|p/p_{cr}| = 0.6$ without absorption and $\gamma = 0$; curve 2, $|p/p_{cr}| = 0.4$ without absorption; curve 3, $|p/p_{cr}| = 0.6$ with $\gamma = 0.24$ ($\alpha = 1$ dB/km); curve 4, $|p/p_{cr}| = 0.4$ with $\gamma = 0.24$.

where $p_j = \beta L I_{j0} \exp\{(-1)^j \gamma[(j-1) - \xi]\}$ and $d = \mu v_g^2/(2L)$ is the normalized dispersion. Here p_j's are the normalized pumping intensities of the two beams (note that p_j's can have a negative sign depending on the nonlinearity β). We consider here the simplest case whereby both counterpropagating waves have equal incidence intensities, $p_1(\xi = 0) = p_2(\xi = 1) = p$. Since the only situation of interest is that in which over the total length of interaction L the pumping is almost undepleted, i.e., $\alpha L = \gamma \ll 1$, one can use an approximation of $p_1 \simeq p_2 \simeq$ const. ($\equiv p$), which is most adequate when the pumping is significantly below the threshold of instability p_{cr} (see below), $p < p_{cr} \exp(-\gamma)$.

The boundary conditions for the pump–probe configuration are $A_{1,1}(\xi = 0) = C =$ const. and $A_{2,1}(\xi = 0) = A_{1,2}(\xi = 1) = A_{2,2}(\xi = 1) = 0$. The numerical results from solving Eq. (3) with these boundary conditions are shown in Figs. 1–3. The intensity amplification $G_1 = |A_{1,1}(\xi = 1)/A_{1,1}(\xi = 0)|^2$ versus the normalized detuning frequency $\delta\Omega$ under various pumping and absorption conditions is plotted in Fig. 1 for a 1-km Ge-doped silica-core fiber at the 1.55-μm wavelength region with group-velocity dispersion $D(k) = 6.5 \times 10^{-3}$ [note that $\mu = -D(k)/(kc^2)$ and the corresponding $d = 8.02 \times 10^{-13}$], a refractive index of 1.44, and $n_2 = 3.2 \times 10^{-16}$ cm^2/W.[9] One can see that the gain G_1 with $\gamma = 0.24$ (corresponding to $\alpha = 1$ dB/km in terms of practical units) is slightly lower than that of the lossless case. The nonlinear fiber example shown has high gain [as much as 10^4 and 10^6 for pumping at 40% and 60% of the normalized threshold intensity, respectively, for instability $p_{cr} = 12.1$ (=9.3 MW/cm^2 in real units)[7]] and is broadband (of the order of terahertz for pumping at 40% and 60% of p_{cr}).

To gain further insight, it is instructive to have an analytical approximation for the gain spectrum. For small dispersion $|d| \ll 1$ (which is always valid for typical situations), the gain G_1 can be represented approximately as

$$G_1 = \left\{\cosh^2(\delta\Omega x) - d^2(|p/d| - \delta\Omega^2)^2 \times \left[\frac{\sinh(\delta\Omega x)}{\delta\Omega x}\right]^2\right\} \frac{\exp(-\gamma)}{D(p, d, \gamma, \delta\Omega)}, \tag{4}$$

where $x = [2|dp| - (d\delta\Omega)^2]^{1/2}$. D is the determinant defining the threshold of instability[7]; $D \simeq 1$ if the pumping p is sufficiently below the threshold of instability p_{cr}, which is true for most cases ($D \rightarrow 0$ when $p \rightarrow p_{cr}$ at each particular detuning frequency $\delta\Omega$). The gain G_1 at small detuning frequencies, $\delta\Omega \ll (\delta\Omega)_0 = (8|pd|)^{-1/2} \ln(1 + p)$, is due to the dispersionless four-wave mixing. However, beyond $(\delta\Omega)_0$ it is mostly due to the dispersion-related process. In the latter case, $G_1 \simeq \exp\{2\delta\Omega[2|dp| - (d\delta\Omega)^2]^{1/2} - \gamma\}/2$.

To show the variation of gain with normalized pumping intensity $|p|$ and absorption, we plot the maximum gain $(G_1)_{max}$ versus $|p/p_{cr}|$ for different absorption coefficients γ in Fig. 2. According to our calculations, the reduction in $(G_1)_{max}$ is negligible for small absorption, and $(G_1)_{max}$ increases exponentially with p as shown in Fig. 2. Since the separation between curves with different γ is small, the enlarged inset in Fig. 2 shows individual curves for various γ

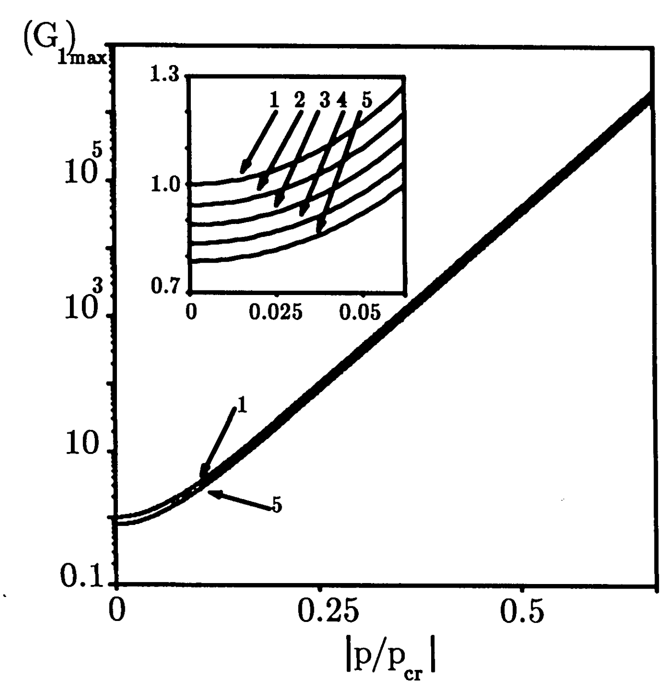

Fig. 2. Amplification $(G_1)_{max}$ versus normalized pumping intensity $|p/p_{cr}|$ for the 1-km-long Ge-doped silica fiber in Fig. 1. Inset: the enlargement around the origin shows individual curves for various absorption coefficients γ (or α). Curve 1, $\gamma = 0$ ($\alpha = 0$); curve 2, $\gamma = 0.06$ ($\alpha = 0.25$ dB/km); curve 3, $\gamma = 0.12$ ($\alpha = 0.5$ dB/km); curve 4, $\gamma = 0.18$ ($\alpha = 0.75$ dB/km); curve 5, $\gamma = 0.24$ ($\alpha = 1$ dB/km).

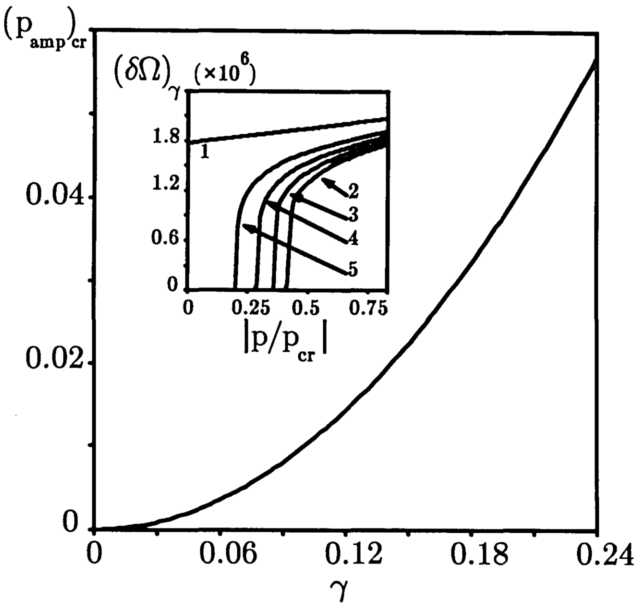

Fig. 3. Threshold of amplification $(p_{amp})_{cr}$ versus the normalized absorption coefficient γ for the 1-km-long Ge-doped silica fiber in Fig. 1 with different absorption coefficients γ. Inset: normalized bandwidth of amplification $(\delta\Omega)_\gamma$ versus normalized pumping intensity $|p/p_{cr}|$. Curves 1–5 correspond to the same absorption coefficients γ as in Fig. 2.

near the origin. The maximum possible gain $(G_1)_{max}$ can be found from Eq. (4) with the assumption $D \simeq 1$,

$$(G_1)_{max} = \exp(-\gamma)\cosh^2 p. \tag{5}$$

G_1 achieves its maximum value, $(G_1)_{max}$, at $(\delta\Omega)_{opt} \simeq (p/d)^{1/2}$.

Although the small absorption results only in a slight decrease in amplification when the pumping intensity is sufficiently large, it becomes crucial for low-intensity pumping. In particular, it results in the existence of a threshold requirement for pumping intensity to provide $G_1 > 1$. We define this threshold $(p_{amp})_{cr}$ as the intensity at which the maximum gain is unity, i.e., as the minimum pumping intensity required to overcome absorption at $\delta\Omega = (\delta\Omega)_{opt}$. From Eq. (5) it is given by

$$(p_{amp})_{cr} = \ln\{\exp(\gamma/2) + [\exp(\gamma) - 1]^{1/2}\} \tag{6}$$

(see Fig. 3). Equation (6) gives $(p_{amp})_{cr} = 0.35$ (270 kW/cm^2 in real units) for a nonlinear fiber with a normalized absorption coefficient $\gamma = 0.12$ (corresponding to $\alpha = 0.5$ dB/km). For a typical fiber with an effective area of 50 μm,[9] the threshold pumping power for amplification is 134 mW, which is 4 to 5 orders of magnitude below the damage threshold of a fiber[9] and 2 orders of magnitude below the threshold of instability p_{cr}.[7]

From Eq. (4) one can note that there is a certain normalized cutoff detuning frequency $(\delta\Omega)_{cr}$ beyond which $G_1 \simeq 1$. For the lossless case, the analytical solution for $(\delta\Omega)_{cr}$ can be readily obtained using Eq. (4) with the condition $G_1 = 1$,

$$(\delta\Omega)_{cr} = \{[|p| + (|p|^2 + \pi^2)^{1/2}]/|d|\}^{1/2}, \tag{7}$$

i.e., for small pumping, $|p| \ll \pi$, there is a nonvanishing limit for cutoff frequency $(\delta\Omega)_{cf} \rightarrow (\pi/|d|)^{1/2}$ (see Fig. 3). In a lossless fiber, $(\delta\Omega)_{cr}$ is the bandwidth of amplification $(\delta\Omega)_\gamma$, defined as the frequency span at which $G_1 > 1$. When the linear absorption is included, G_1 at $\delta\Omega \simeq 0$ may decrease to below unity, and the bandwidth of amplification $(\delta\Omega)_\gamma$ narrows as pumping $|p|$ is reduced to $(p_{amp})_{cr}$. This situation is illustrated in the inset of Fig. 3, where $(\delta\Omega)_\gamma$ versus $|p/p_{cr}|$ is plotted. As long as the pumping intensity $|p|$ significantly exceeds the threshold of amplification $(p_{amp})_{cr}$ for a fixed absorption, $(\delta\Omega)_\gamma$ is almost the same as $(\delta\Omega)_{cr}$ for the lossless situation.

In conclusion, we have shown that linear frequency dispersion together with the third-order nonlinearity can provide large, broadband amplification in an optical fiber pumped by counterpropagating waves. This scheme may provide a promising method of optical amplification in low-loss fibers.

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