

Modulation-Induced Multitransparency, Inhibition of Dynamics, and High-Order Frequency Mixing in a Periodically Driven Two-Level Atomic System

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Abstract – At certain (multiple) resonant conditions, a periodically modulated field driving a two-level system can suppress the dynamics of both its population and polarization and induce self-transparency. Outside of these narrow resonances, the system exhibits a high-order frequency mixing with very high conversion efficiency and the spectral plateau giving rise to a soliton-like train as the field propagates. Our computer simulations show a universal effect of full revival of the field after certain propagation distances.

Electromagnetically induced transparency (EIT) of three-level atoms in a strong laser pulse [1] is attracting a growing interest motivated by, e.g., nonlinear frequency transformation [2]. Its successful exploration provides stimulating background for the search for other resonant field-induced nonlinear effects. In this paper, we demonstrate the feasibility of a modulation-induced inhibition of dynamics (MID), self-transparency, and very high order frequency mixing (HFM) in an even simpler, two-level system (TLS) subjected to a periodically modulated laser field. In contrast to self-induced transparency (SIT) in 2π solitons [3] characterized by the dramatic dynamics of population, in MID the TLS dynamics (i.e., the oscillations of both the population and polarization) is almost fully suppressed. For a sinusoidal modulation, the conditions for MID coincide with those for the resonances occurring if the frequency of modulation Ω_M is close to a subharmonic of the averaged Rabi frequency $\overline{\Omega_R}$ [4 - 6] (see also references therein to earlier work). It has also been shown [7] that similar resonances in TLS may occur even when the system is driven by a cw (nonmodulated!) field strong enough to “superdress” TLS by having Ω_R exceed the TLS unperturbed frequency ω_0 . The work [4 - 6] concentrated on the spontaneous radiation, i.e., fluorescence, which peaks at these resonances, where the population difference δ vanishes. We show that, at these resonances, the TLS driven polarization p vanishes too, resulting, together with vanishing δ , in the collapse of the “Rabi sphere” (or a “Rabi circle” in the case considered here). Thus, even as the spectral components of the driving field are strongly off the resonance, the field-induced Rabi-splitting brings TLS at the MID points into exact resonance with the field (similarly to [7]), which results in a strong saturation, reminiscent of that in a cw driving. The important point to note is that, in contrast to the cw saturation, MID results in a self-transparency for the entire spectrum of the incident field (which could be very broad), allowing

its propagation without dispersion suppressed now over the spectrum.

In a relaxationless TLS, if its dynamics is characterized “globally” by a Rabi radius, C_R , with $C_R^2 = \delta^2 + p^2$, one finds that C_R is a (so-called Casimir) invariant (=1) independent of the driving field. However, in the modulation-driven TLS with relaxation (even when it is asymptotically vanishing), the radius C_R , although remaining constant in time, can be less than unity even outside of MID resonances (and vanish at them), depending on the modulation parameters. This inhibited dynamics occurs to be related to the time-averaged population difference, $\overline{\delta}$, where “bar” designates averaging over the modulation period, i.e., $\overline{w(\eta)} = (2\pi)^{-1} \int_{-\pi}^{\pi} w(\eta) d\eta$, where $\eta = \Omega_M t$ and $w(\eta)$ is a generic 2π -periodic function. For the so-called full modulation, a very simple relation is found: $C_R^2 = \overline{\delta} = \overline{\cos \Phi}$, where Φ is a Rabi phase. This result is valid for any periodic (but not necessarily sinusoidal) modulation. We found an exact analytic solution (in closed form, instead of infinite extensions or continued fractions as in [4 - 6, 8]) for the sinusoidal modulation of the TLS with an arbitrary strong relaxation, determined the invariant in “vanishing relaxation” limit, and then generalized the result to an arbitrary periodic modulation.

The MID resonances occur only at certain ratios $\overline{\Omega_R}/\Omega_M$ or in their close vicinity. Probably even more interesting effects occur outside of them, where the periodic modulation induces dramatic TLS dynamics that can give rise to HFM. We found an interesting parallel between HFM and recently discovered high harmonic generation (HHG) by very intense optical fields in rare gasses [9], whereby, beginning at some harmonic order, the harmonics have the same order of amplitude up to some cutoff frequency, forming a spectral plateau followed by a rapid fall-off. One of the simplest models of

HHG pertinent to our subject is an overdriven TLS [10, 7] allowing an analytical solution [7] for the cutoff frequency. We show that a TLS-generated HFM spectrum, $\omega_n = \omega \pm (2n + 1)\Omega_M$, $n = 0, 1, 2, 3, \dots$, features a similar plateau. Note that, in contrast to HHG [7, 10], HFM exists within the perturbation theory, i.e., when $\Omega_R \ll \omega_0$ and the rotating field approximation is still valid. Furthermore, unlike HHG, during the propagation, HFM demonstrates large conversion efficiency, with the fundamental components almost vanishing at some points and their energy transversed into higher spectral components. This process gives rise to a train of soliton-like pulses, with the energy redistributed to produce short spikes of high peak intensity. Thus, HFM can be used for coherent generation of new frequencies (e.g., for nonlinear spectroscopy) and of short pulses of high intensity.

Consider a driving laser field, $\mathcal{E}_L = E(t)\cos\omega t$, whose (real) amplitude envelope, $E(t)$, is periodically modulated, $E(t) = E(t + 2\pi/\Omega_M)$ ($\Omega_M \ll \omega$). The most pronounced MID occurs (i) at the exact resonance, $\omega = \omega_0$, and if (ii) $E(t)$ is an "odd-cycle" function, $E(t) = -E(t + \pi/\Omega_M)$, with its spectrum having only odd harmonics of Ω_M , and $\bar{E} = 0$, implying the so-called full amplitude modulation, with the component at ω vanishing (a "suppressed carrier"). The simplest is sinusoidal modulation, $E(t) = E_M \cos(\Omega_M t)$, $E_M = \text{const}$, resulting in only two spectral components, $(E_M/2)\cos[(\omega \pm \Omega_M)T]$, separated by $2\Omega_M$, as, e.g., with two lasers, or an atomic beam traveling through a standing laser wave, such that the atoms see two frequencies spaced by double Doppler shift. We assume $\Omega_R \ll \omega_0$, where $\Omega_R(t) = \mu E/\hbar$ is the Rabi frequency with $\mu = e d_{12}$ being the TLS dipole moment, and use a density matrix whose diagonal elements are the populations per atom at the ground (ρ_{11}) and excited (ρ_{22}) levels ($\rho_{11} + \rho_{22} = 1$); the nondiagonal elements, $\rho_{12} = \rho_{21}^*$, are related to the induced polarization $\mathbf{P} = \mu(\rho_{12} + \rho_{21})$. The population difference, $\Delta \equiv \rho_{11} - \rho_{22}$, has an absolute maximum, $|\Delta| = \Delta_B$, at the Boltzmann thermal equilibrium (in the cases of interest, $\Delta_B = 1$). Writing ρ_{21} in the form $\rho_{21} = -ip(\Delta_B/2)\exp(-i\omega_0 t)$, one can show that, at $\omega = \omega_0$, its slow envelope p , similarly to $E(t)$, is real. Using normalized variables: p , $\delta \equiv \Delta/\Delta_B$, relaxation rates $\gamma_r \equiv (\tau\Omega_M)^{-1}$ and $\gamma_T \equiv (T\Omega_M)^{-1}$, where τ and T are life time and phase coherence time respectively, $T \leq 2\tau$, field envelope $R \equiv \mu \cdot E/\hbar\Omega_M$, and time $\eta \equiv \Omega_M t$, the Bloch equations [11, 12] for the TLS dynamics are written as

$$\begin{aligned} d\delta/d\eta + \gamma_r(\delta - 1) &= R(\eta)p; \\ dp/d\eta + \gamma_T p &= -R(\eta)\delta. \end{aligned} \quad (1)$$

By introducing the Rabi phase, $\Phi(\eta) \equiv \int_{-\infty}^{\eta} R(s)ds$, they are readily solved in quadratures even when $R(\eta)$ is an

arbitrary function of η ; if both the relaxation rates are the same, $\gamma_r = \gamma_T = \gamma$ [12]:

$$\delta(\eta) = \int_{-\infty}^{\eta} \Gamma \cos \Delta\Phi d\eta'; \quad p(\eta) = - \int_{-\infty}^{\eta} \Gamma \sin \Delta\Phi d\eta', \quad (2)$$

where $\Delta\Phi \equiv \Phi(\eta) - \Phi(\eta')$ and $\Gamma \equiv \gamma e^{\gamma(\eta' - \eta)}$. On the other hand, in a relaxationless TLS, as is well known,

$$\delta_{\text{rls}}(\eta) = C_{\text{rls}} \cos \Phi(\eta), \quad p_{\text{rls}}(\eta) = -C_{\text{rls}} \sin \Phi(\eta), \quad (3)$$

where $C_{\text{rls}} = \text{const}$ is the Rabi radius. The limit in the integral in $\Phi(\eta)$ is chosen such that at $\eta \rightarrow -\infty$ (assuming $R \rightarrow 0$) the atom is at the ground level (i.e. $\rho_{11} = 1$), and thus $\cos \Phi(-\infty) = C_{\text{rls}} = 1$. Here, $C_{\text{rls}}^2 = \delta^2 + p^2 = 1$ is an invariant directly related to the conservation of total population and independent of driving $R(\eta)$. Indeed, using amplitudes a_1 and a_2 of the wave functions of the ground and excited levels, respectively, such that

$\rho_{jk} = a_j a_k^* \exp[i(\omega_k - \omega_j)t]$, with $\omega_2 - \omega_1 = \omega_0$, one rewrites the invariant in the form $(|a_1|^2 - |a_2|^2)^2 + 4|a_1 a_2|^2 =$

$(\rho_{11} + \rho_{22})^2$. Since $\rho_{11} + \rho_{22} = 1$, $C_{\text{rls}}^2 = 1$. For $\gamma \neq 0$, the Rabi radius is still = 1 for a short pulse of a finite length ($\ll T, \tau$), with $E(t) \rightarrow 0$ as $|t| \rightarrow \infty$ (e.g. 2π -soliton). (Even when $\gamma \neq 0$, (2) can be evaluated analytically for many classes of functions $R(\eta)$ [12].) However, when the driving is much longer than T, τ , one has to solve the Bloch equations for a finite relaxation (with no memory of the initial conditions kept when $\eta \gg \gamma^{-1}$) and then find the solution in the limit $\gamma \rightarrow 0$. The Rabi radius, C_R , evaluated now by using (2) and seeking the limit $\delta^2 + p^2 \rightarrow C_R^2$ as $\gamma \rightarrow 0$, in the general case can be smaller than unity and even reach zero. The most trivial example of this is a resonant cw driving field, $E(t) = \text{const}$, $\omega = \omega_0$. Equation (2) yields then a familiar result $\delta = 1/[1 + (\Omega_R T)^2]$; $p = -\Omega_R T \delta$; thus, as $T^{-1} \rightarrow 0$, $C \rightarrow C_R = 0$. [Note that here, even for $\gamma \neq 0$, $\delta^2 + p^2 = \delta$; compare with (9).] For the periodically modulated driving with $\gamma\eta \gg 1$ (when no memory of the initial conditions is left), the Rabi radius can readily be found using the fact that $\sin \Delta\Phi$ and $\cos \Delta\Phi$ in (2) are fast varying functions as compared to $\exp(\gamma\eta')$ when $\gamma \rightarrow 0$. The results are especially simple [see (9) below] for an arbitrary odd-cycle modulation, when $\overline{\Phi(\eta)} = \overline{\sin \Phi(\eta)} = 0$. It is instructive, however, to first solve (2) analytically in closed form (in place of infinite extensions or continuous fractions [4 - 6, 8]), valid for any γ , for at least a sinusoidal modulation, and then find the solution in the limit $\gamma \rightarrow 0$. For $R(\eta) = M \cos \eta$, where $M = \mu E_M/\hbar\Omega_M$, solving (2) in Fourier extensions $\delta(\eta) = \delta_0 + (\sum_{n=1}^{\infty} \delta_n e^{2in\eta} + \text{c.c.})$ and

$p(\eta) = i[\sum_{n=1}^{\infty} p_n e^{i(2n-1)\eta} - \text{c.c.}]$, we derive from (3) the recursion relations for δ_n and p_n for arbitrary T and τ :

$$p_n + p_{n+1} = 2\delta_n(2n - i\gamma\tau)/M, \quad (4a)$$

$$\delta_{n-1} + \delta_n = 2p_n(2n - 1 - i\gamma\tau)/M, \quad (4b)$$

except for $n = 0$, where

$$\gamma_i(\delta_0 - 1) = -M \cdot \text{Im}(p_1). \quad (5)$$

(For numerical simulation purposes, one can further reduce (4) to one variable, say, δ_n [13].) If $T = \tau$, using a familiar formula for a generic Bessel function, $B_{\nu+1}(M) + B_{\nu-1}(M) = (2\nu/M)B_{\nu}(M)$ and stipulating that δ_n and p_n are finite at $M = 0$ and $M \rightarrow \infty$, we found an analytical solution for the coefficients p_n and δ_n valid for any M and γ :

$$\delta_n = CJ_{2n-i\gamma}(M); \quad p_n = CJ_{2n-1-i\gamma}(M), \quad (6a)$$

with

$$C = \gamma\pi J_{\gamma}(M)/\sinh(\gamma\pi), \quad (6b)$$

where $J_{\nu}(x)$ is a Bessel function of the first order and $C = \text{const}$ is obtained from (5) by using another familiar formula, $J_{\nu}(M)J_{1-\nu}(M) + J_{-\nu}(M)J_{\nu-1}(M) = 2\sin(\nu\pi)/\pi M$. In the limit $\gamma \rightarrow 0$, the solution to (2) and (6) tends to a relaxationless-like solution, (3), but with a new Rabi radius, C_R , instead of C_{rls} :

$$|C|^2 \rightarrow C_R^2(M) = \delta_0(M) = J_0^2(M). \quad (7)$$

We can see now that the dc component δ_0 of population difference, according to (7), is always non-negative (similarly to cw driving), $\delta_0(M)|_{\gamma \rightarrow 0} = J_0^2(M) \geq 0$, whereas in a relaxationless TLS (3) $[\delta_0(M)]_{\text{sa}} = J_0(M)$, such that $\overline{\delta_{\text{rls}}}$ can be either positive or negative [note also that $(\delta_0)_{\gamma \rightarrow 0} = \overline{\delta_{\text{rls}}}^2$]. Also, we have now $0 \leq C_R^2 \leq 1$, in contrast to a relaxationless TLS, with $C_{\text{rls}}^2 = 1$. Finally, when $J_0(M) = 0$, C_R vanishes along with δ_0 , resulting in the MID mode [14]. Thus, the solutions for small γ 's are (almost) the same (7) when $\eta \rightarrow \infty$. [It is obvious, however, that the transient behavior of TLS whereby, e.g., the modulation amplitude slowly rises from zero (at $\eta = -\infty$) to $\text{const} = M$, will strongly depend on γ .] The above features are retained even for an arbitrary ratio T/τ , although the radius C_R outside of MID depends now on T/τ even as $T, \tau \rightarrow \infty$ and can be found using [6] as

$$\delta_0 = 2(T/\tau)J_0^2(M)/[1 + T/\tau + J_0(2M)(T/\tau - 1)], \quad (8a)$$

$$C_R = \delta_0(M)/J_0(M), \quad (8b)$$

in particular, for the radiation-related relaxation, $T = 2\tau$, one finds $C_R = 4J_0(M)/[3 + J_0(2M)]$. In the presence of inhomogeneous broadening $\Delta\omega_{\text{inh}}$ (not considered here), one can still expect, similarly to EIT [1, 2], the MID effect if $\Omega_M, \Omega_R \gg \Delta\omega_{\text{inh}}$.

A simple generalization of (7) is found for any periodic (and not just sinusoidal) envelope $R(\eta)$, provided it is an odd-cycle function. Then, if starting from $\bar{R} = 0$ at $\eta \rightarrow -\infty$, the modulation slowly reaches a steady state, and $\Phi(\eta)$ in (3) becomes also an odd-cycle function along with $p_{\text{rls}}(\eta)$, i.e., $\overline{\Phi(\eta)} = \overline{p_{\text{rls}}(\eta)} = 0$. Evaluating a time averaged δ of a relaxationless TLS [i.e., δ_{rls} with $C_{\text{rls}} = 1$ in (3)] as $\overline{\delta_{\text{rls}}} = \overline{\cos \Phi(\eta)}$, we find that the solution to $\delta(\eta)$ and $p(\eta)$ (2) in the density matrix description with $T = \tau = 1/\gamma$ in the limit $\gamma \rightarrow 0$ is given by the relaxationless TLS response (3) in which, instead of $C_{\text{rls}} = 1$, one has to assume now

$$C_R = \overline{\delta_{\text{rls}}} \equiv \overline{\cos \Phi(\eta)}, \quad \text{or} \quad \delta_0 \equiv \overline{\delta(\eta)} = C_R^2 = \overline{\delta_{\text{rls}}}^2. \quad (9)$$

One can verify this also by multiplying the 1st and 2nd equations in (1) by δ and p , respectively, summing them up: $(1/2)d(p^2 + \delta^2)/d\eta + \gamma(p^2 + \delta^2) = \gamma\delta$, and averaging: $\overline{p^2} + \overline{\delta^2} = \delta_0$. In the limit $\gamma \rightarrow 0$, using (2), we obtain $C_R^2 = \delta_0$, and also $\delta_0 = C_R \overline{\delta_{\text{rls}}}$, i.e., (9). The properties due to a sinusoidal modulation can be generalized now: $\overline{\delta(\eta)}$ is always non-negative; C_R can be less than unity; at certain conditions both of them vanish resulting in MID. Consider, e.g., a modulation by either a "sharpened" or a "flattened" sinusoid (see insert in Fig. 1):

$$R(\eta) = MB \cos \eta (1 + A \cos^2 \eta)^{-1/2}, \quad (10)$$

where $A > -1$ controls the distortion of the sinusoid and B conserves an averaged Rabi frequency, $\bar{R} = 2M/\pi$: $B = \sqrt{A}/\sin^{-1} \bar{A}$ [with $\Phi = M \sin^{-1}(\bar{A} \sin \eta)$] if $A > 0$, and $B = \sqrt{|A|}/\sinh^{-1} \bar{A}$ [with $\Phi = M \sinh^{-1}(\bar{A} \sin \eta)$] if $A < 0$, where $\bar{A} = \sqrt{|A|}/(1+A)$. Here, $A = 0$ gives a sinusoidal modulation with $C_R = J_0(M)$. The limit $A \rightarrow \infty$ gives a rectangular modulation, with $R = \text{const}$ switching sign at $\eta = \eta_0 + m\pi$ (or with its phase hopping there by π); the solution here is $C_R = \sin(M)/M$ with the MID points $M_{\text{MID}} = \pi, 2\pi, 3\pi, \dots$. The limit $A \rightarrow -1$ gives a periodic train of δ -functions, with $C_R = \cos(M)$ and $M_{\text{MID}} = \pi/2, 3\pi/2, 5\pi/2, \dots$. $C_R^2 = \delta_0$ vs. M for various envelopes ($A = 0; -0.988; 20$) is shown in Fig. 1, with clearly pronounced MID points for all the cases. In all the cases, the MID points are spaced by $\Delta M = \pi$ (same as in the overdriven TLS [7]) indicating resonances between odd harmonics of the driving frequency, $(2n-1)\Omega_M$, and the averaged Rabi frequency, $|\overline{\Omega_R}| = 2|\mu E_M|/\hbar\pi = 2M\Omega_M/\pi$ (they are usually viewed as subharmonics of $|\overline{\Omega_R}|$, [4-6]). Thus, MID is related to the coherent interference between multiphoton HFM and oscillations at the (averaged) Rabi frequency.

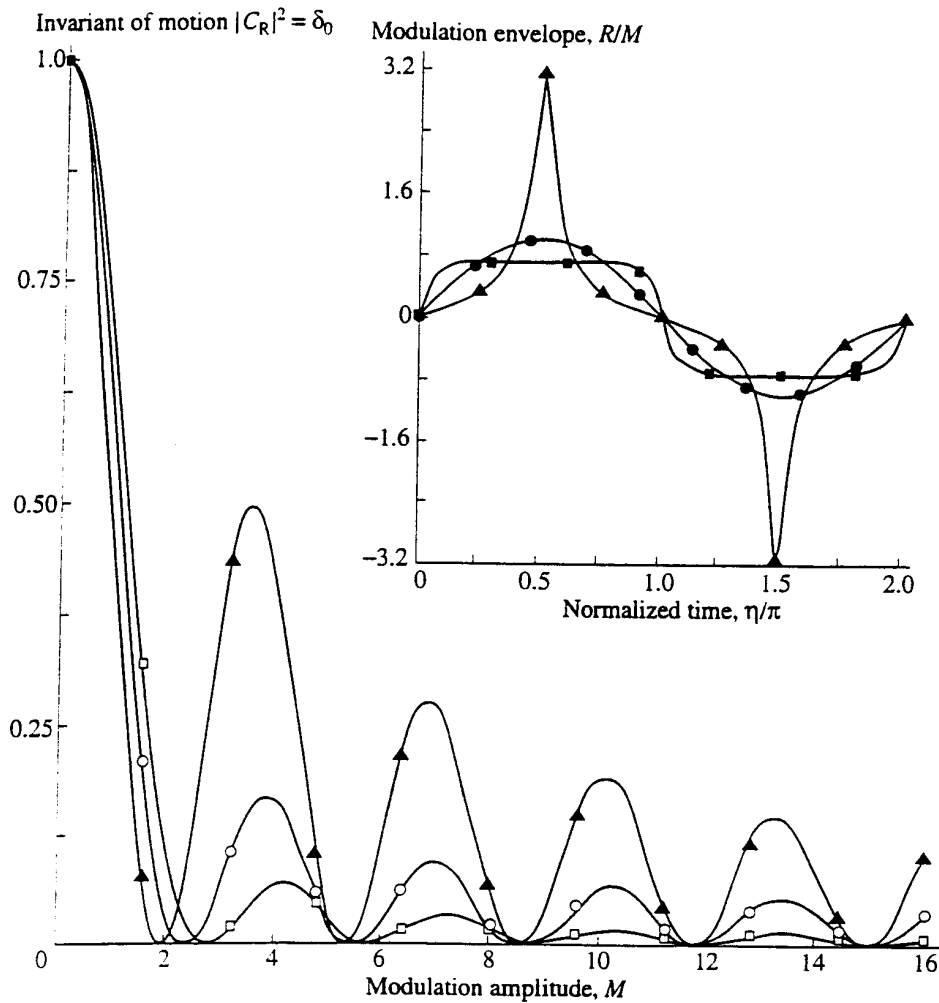


Fig. 1. The invariant of motion, $C_R^2 = \delta_0$ (C_R - Rabi radius), vs. the amplitude of modulation, M , for the modulation in the form of regular sinusoid (O), "flattened" (□), and "sharpened" sinusoid (Δ). The respective modulation envelopes are depicted in the insert.

An ergodic-like correspondence between a relaxationless TLS with $C_R^2 = 1$ and a TLS with $\gamma \rightarrow 0$ and $C_R^2 \leq 1$ is found by introducing an ensemble averaging of the former with a random phase ϕ in (3). $\bar{\Phi}(\eta) = \Phi(\eta) + \phi$. Choosing ϕ such that its probability distribution is a symmetric function of ϕ , i.e., $\langle \phi \rangle = 0$ and $\langle \sin \phi \rangle = 0$, where $\langle \rangle$ designate ensemble averaging, we have $\delta(\eta) = \langle \delta_{rk}(\eta) \rangle = \langle \cos \phi \rangle \cos \Phi(\eta)$; $p(\eta) = \langle p_{rk}(\eta) \rangle = -\langle \cos \phi \rangle \sin \Phi(\eta)$; i.e., $\langle \cos \phi \rangle^2 = C_R^2 \leq 1$. The correct result, (9), corresponds then to $\langle \cos \phi \rangle = \overline{\cos \Phi}$.

Considering now the propagation of the plane wave in the z -axis, assuming $\mathcal{E} = \text{Re} \{ \mathbf{E}(z, t) \exp[i(k_0 z - \omega_0 t)] \}$ and $p_{21} = -i(\Delta_B/2)p(z, t) \exp[i(k_0 z - \omega_0 t)]$, where $k_0 = \omega_0 n_0 / c$, n_0 is a background refractive index, and using the normalized distance $\zeta \equiv z k_M$ (with $k_M = \Omega_M / v_0$, where v_0 is a nonresonant group velocity), we obtain

the Maxwell equation for the envelope $R(\eta, \zeta)$ as

$$\partial R / \partial \zeta + \partial R / \partial \eta = Qp, \tag{11}$$

where $Q = \alpha \Delta_B N d_{12}^2 \lambda_0 (\omega_0 / \Omega_M)^2$ with N being the atomic density, $\alpha = e^2 / \hbar c = 1/137$ - the fine structure constant, and $\lambda_0 = 2\pi v_0 / \omega_0$. Equation (11) is coupled to the Bloch equations (1) with $d/d\eta$ replaced by $\partial/\partial\eta$. In a MID mode, $p \approx 0$ and TLS becomes "invisible" to the wave. Note that the parameters of the envelope required for that depend only on atomic characteristics and are density independent. In the general case, we found that, in the limit $\gamma \rightarrow 0$, the Maxwell-Bloch equations, (1) and (11), have two first invariants signifying separate conservations of EM energy and atomic system oscillation energy, respectively, as the wave propagates:

$$\bar{R}^2(\zeta) = \text{inv}, \quad \delta_0(\zeta) = C_R^2 = \bar{\delta}_{rk}^2(\zeta) = \text{inv}. \tag{12}$$

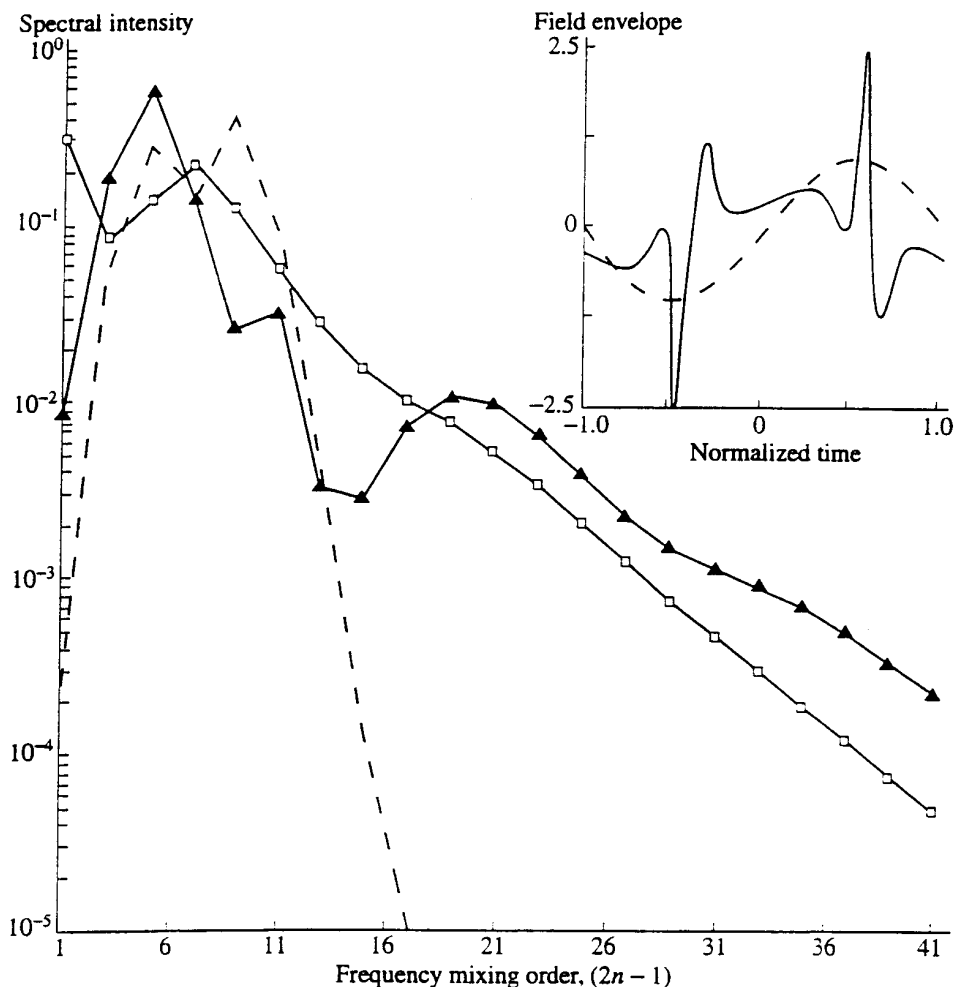


Fig. 2. The HFM spectra (spectral intensities normalized to their full sum vs. the HFM order $2n - 1$) of the polarization p (dashed) at the boundary ($\zeta = 0$) and of p (Δ) and the field envelope R (\square) at $\zeta = 2348/Q$ (the total spatial cycle is $\zeta_c = 4950/Q$), when $M = 10.2$. Insert: R vs. normalized time, τ/π , at $\zeta = 0$ (dashed) and at $\zeta = 2348/Q$ (solid).

(The "area theorem" for $\Phi = \int_{-\infty}^{\infty} R(\zeta, \eta) d\eta$ is reduced now to $\overline{\Phi}(\xi) = 0$, with $\Phi(\eta, \zeta)$, $R(\eta, \zeta)$, and $p(\eta, \zeta)$ remaining odd-cycle functions of η .) The variables Φ , ζ , and $\theta = \eta - \zeta$ transform the Maxwell-Bloch equations (1) and (11) in the limit $\gamma \rightarrow 0$ to a sine-Gordon equation,

$$\frac{\partial^2 \Phi}{\partial \theta \partial \zeta} + q \sin \Phi = 0, \quad q \equiv Q \bar{\delta}_{rls} = \text{const}, \quad (13)$$

with $\bar{\delta}_{rls}$ being already an invariant determined by the boundary condition. (In addition to $\overline{R^2}(\zeta) = \text{inv}$ (12), (13) has an infinite number of invariants [15].) The linear case ($\Phi \ll \pi$, $\bar{\delta}_{rls} = 1$) results in the expected solution $\Phi = \Phi_0 \sin[\eta - (1 - Q)\zeta]$ with the strong dispersion of its group velocity $v_{gr}(\Omega_M) = v_0/[1 - Q(\Omega_M)]$ (note that $Q \propto \Omega_M^{-2}$). At the MID mode, by its definition, $\bar{\delta}_{rls} = 0$,

so that $\partial \Phi / \partial \zeta = 0$, and the solution is any function $\Phi(\theta)$ satisfying the condition $\overline{\cos \Phi} = 0$. Hence, the MID mode with an arbitrary profile (as well as spectrum) propagates without any distortion, dispersion, or absorption with the nonresonant group velocity v_0 since the dispersion built into TLS is now suppressed along with its polarization.

If $\overline{\cos \Phi}(\zeta = 0) \neq 0$, the system can still support modes with conserved profile and group velocity, which essentially are an "extension" of SIT-solitons into the nonlinear wave train: $\Phi = F(\theta + v\zeta)$, where $v = \text{const} \equiv 1 - v_0/\bar{v}$ is due to the new group velocity of the train, $\bar{v} (< v_0)$. \bar{v} is determined by the boundary conditions (or the peak intensity of the wave), and function $F(\xi)$ - by the "pendulum" equation $d^2 F / d\xi^2 + (Q \bar{\delta}_{rls} / v) \sin F = 0$. The difference between this "SIT"-mode and the MID-mode is that the SIT-mode can have only one profile for each v , with v

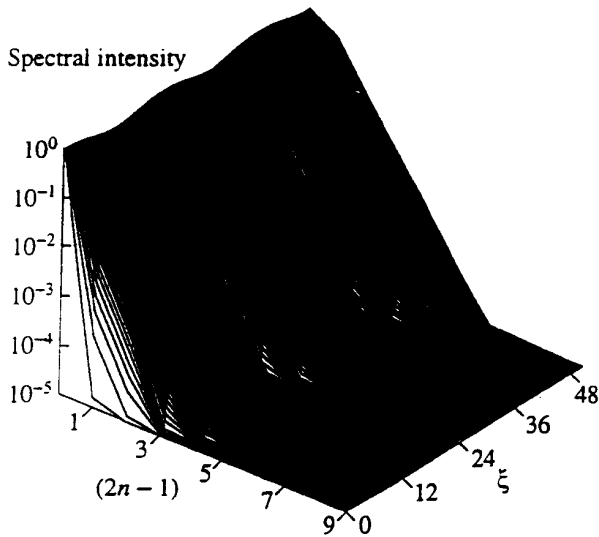


Fig. 3. The spatial dynamics of HFM: spectral intensities, $|R_{2n-1}|^2$ of the $(2n-1)$ th harmonics of the envelope R vs. the normalized propagation distance ζ for $M = 3.5$.

affected by the intensity and atomic density $N(\propto Q)$ and both population difference δ and polarization p oscillating dramatically, whereas in the MID-mode, δ and p are fully suppressed as $\gamma \rightarrow 0$ and $\nu = 0$. [The whole notion of the well-known SIT-modes has now to be revised in view of factor $\bar{\delta}_{\text{ris}}$ in (13). We found that, as the peak intensity of SIT-mode increases, $\bar{\delta}_{\text{ris}} \rightarrow 0$, and such a SIT-mode tends to a MID mode.]

Finally, if the conditions neither for the MID nor SIT modes are satisfied, the envelope undergoes dramatic changes as it propagates, with the amplitudes of the generated HFM components becoming comparable to and even exceeding that of the incidence components. With the sinusoidal modulation at the boundary, the HFM spectrum $\omega \pm (2n-1)\Omega_M$ of the polarization (6a) extends in a plateau-like fashion up to a cutoff point,

$$2n_{\text{cut}} - 1 = \Omega_{\text{cut}} / \Omega_M = \sqrt{4 + M^2}, \quad (14)$$

beyond which it rapidly falls off (see Fig. 2); this is related to the properties of the Bessel function $J_m(M)$ (6a) as function of m for fixed M [the cutoff corresponds to the point at which $J_m(M)$ switches from monotonic to oscillating behavior]. In the HFM-mode, this polarization spectrum is transformed into a plateau-like field spectrum (with a slightly higher cutoff) as the field propagates (Fig. 2). In the time domain, HFM gives rise to the train of soliton-like pulses [16] (see insert in Fig. 2), with the individual length of each being

$$\tau_M = \pi / (\Omega_M \sqrt{4 + M^2}). \quad (15)$$

(If $\lambda_0 = 1 \mu\text{m}$, $\Omega_M / \omega_0 = 1.5\%$, $M = 10$, and $\tau_M = 100$ fs.) Our computer simulations using (11) showed though

that, beyond the cutoff, the field spectrum in the HFM mode falls off as $|R_m|^2 \propto e^{-Sm}$, where m is the HFM order and $S(M)$ is some constant (see Figs. 2 and 3), whereas the polarization spectrum at the boundary ($\zeta = 0$) decreases much faster: $|p_m|^2 \propto (eM/2m)^{2m}$. We also found that (for the investigated range, $M \leq 10.5$), after some portion of energy is converted from the lowest spectral field components to the higher ones as the envelope propagates, the inverse process sets in, resulting in the full spatial revival of the field envelope and its spectrum (to precision better than $\sim 0.2\%$, with the invariants (12) conserved to $\sim 10^{-4}$). This means that, at some point $\zeta = \zeta_c$, the field envelope $R(\theta, \zeta_c)$ returns to its original profile $R(\theta, 0)$ and the entire spatial cycle is repeated again; see Fig. 3 for the case $M = 3.5$. This is consistent with the existence of at least some spatially periodic (non-SIT) solutions in the inverse-scattering theory of the sine-Gordon equation [15].

The MID and HFM effects can be observed in a configuration similar to SIT [3] (using now a continuously modulated wave instead of a short pulse) or subharmonic resonance experiments [5] (using now long propagation path and observing the field transformation).

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13. This can be done by writing $p_n = M(\delta_{n-1} + \delta_n)/2(2n - 1 - i\gamma_T)$ and using it in (7a), which results in a recursion relation for $r_n \equiv \delta_n/\delta_{n-1}$: $(r_n^{-1} + 1)/(2n - 1 - i\gamma_T) + (r_{n+1} + 1)/(2n + 1 - i\gamma_T) = 4(2n - i\gamma_T)/M^2$; and δ_0 : $\delta_0^{-1} = 1 + (M^2/2\gamma_T)\text{Im}[(1 + r_1)/(1 - i\gamma_T)]$.
14. When $\gamma \ll 1$, a "residual" δ_0 is evaluated using (4) as $(\delta_0)_{\text{MID}} = Y_0^2(M_{\text{MID}})\gamma^2/4$, where $Y_0(M)$ is a Bessel function of the second order. Comparing this with the saturation due to cw driving, $\mathcal{E} = E_M \cos(\omega t)$ (with the same amplitude E_M and detuning $|\omega - \omega_0| = \Omega_M$ as for modulation), $\delta_{\text{cw}} = (1 + M^2)^{-1}$, one can see that $(\delta_0)_{\text{MID}}/\delta_{\text{cw}} = O(\gamma^2) \ll 1$. For a typical case, $\gamma \sim 10^{-2} - 10^{-3}$, $(\delta_0)_{\text{MID}} \sim 10^{-5} - 10^{-7}$.
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16. A somewhat similar process, albeit on a much faster scale, may result in subfemtosecond pulses in the mode-locked solitons of the cascade stimulated Raman scattering, Kaplan, A.E., 1994, *Phys. Rev. Lett.*, **73**, 1243.