Electromagnetic "Bubbles" and Shock Waves: Unipolar, Nonoscillating EM Solitons

A. E. Kaplan and P. L. Shkolnikov

Department of Electrical and Computer Engineering, The Johns Hopkins University, Baltimore, Maryland 21218

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We show that atomic gases can support solitary pulses of a unipolar electromagnetic field ("EM bubbles") with amplitude up to the atomic field ($\sim 10^9$ V/cm) and duration down to $\sim 10^{-16}$ s. EM bubbles propagate without dispersion, are stable, and are insensitive to the change of gas density. Atomic gasses can also support an EM shock wave forming a precursor of a dc ionizing field.

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Quantum electronics and nonlinear optics operate usually with almost-harmonic EM oscillations modulated by an envelope much longer than a single cycle of the oscillation, with their spectral width substantially smaller than the carrier frequency. Typical examples are 2π solitons in two-level systems (TLS) [1(a)], mode-locked laser pulses [1(b)] due to multimode cavity interaction with laser medium, and optical-fiber solitons [1(c)] due to Kerr nonlinearity. In all of them, slow varying envelope approximations are used in both the propagation (by reducing Maxwell equations to a parabolic partial differential equation) and the material response [rotating-wave approximation (RWA) in constitutive equations]. Many applications, however, in particular the study of atomic physics by means of photoionization, call for short and intense EM pulses of *nonoscillating* nature, having wide spectra that spread from zero frequency. Atomic ionization with almost unipolar "half-cycle pulses" has been of substantial recent interest (see, e.g., [2]). Currently available pulses generated in semiconductor structures are $\sim$400 fs long with the peak field of $\sim$200 kV/cm.

Shorter (10^{-14}–10^{-16} s) and more intense (up to 10^9 V/cm) unipolar pulses might be of great interest for the host of applications. They can be used for a "global" spectroscopic technique based on a shocklike excitation over the entire atomic spectrum, including the normally prohibited transitions. The ionization by a pulse shorter than the orbital period may bridge a gap between photoionization and collisional ionization [2(b)]. Time-domain spectroscopy of dielectrics, semiconductors, and flames [3(a)] with these pulses may expand this method from presently available THz domain [3(b)] to infrared-optical domain. These pulses may be used in time-resolved spectroscopy of transient chemical processes on a fs and sub-fs time scale, e.g., dissociation and autoionization [3(c)], and for quantum control of chemical transformations [3(d)]. One can envision their applications to probing high-density plasmas in tokamaks, pulse-dispersion diagnostic of gases, very high frequency up-conversion due to a large Doppler shift of counterpropagating light backscattered by such a pulse, etc. These pulses may also be used for imaging [3(e)], medical infrared tomography, etc.

Supershort pulses can be generated only by strongly nonlinear processes [4]. Especially significant are *solitary* waves propagating with unchanged shape and length. An exact solitonlike solution for a unipolar pulse in a strongly driven TLS, described by full Maxwell + full Bloch equations, was found quite a while ago [5].

Here we show that such solitons are feasible and natural phenomena for many systems, both quantum and classical. Their length may range from a small fraction of the cycle length of the resonance supporting them to much longer than that cycle, depending on their intensity. We will call them EM bubbles (EMB) to stress their nonenvelope nature. We demonstrate that photoionization imposes an upper limit on the EMB amplitude (typically $\sim 10^9$ V/cm) and a lower limit on its length ($\sim 10^{-16}$ s). The shortest EMB is reached at a certain amplitude; at some larger amplitude, an EMB degenerates into a shock wave.

For a plane EM wave propagating along the z axis, the Maxwell equation for the electric field $\boldsymbol{E}$ is

$$c^2\partial^2\boldsymbol{E}/\partial z^2 - \partial^2\boldsymbol{E}/\partial t^2 = 4\pi\partial^2\boldsymbol{P}/\partial t^2, \qquad (1)$$

where $\boldsymbol{P}$ is polarization. Assuming that an EMB propagates with a constant velocity, βc, introducing retarded variables $\tilde{t} \equiv t - z/\beta c$ and $\tilde{z} = z$, imposing a steady state condition $\partial\boldsymbol{E}/\partial\tilde{z} = \partial\boldsymbol{P}/\partial\tilde{z} = 0$, and stipulating that the EMB has finite energy, i.e., $\boldsymbol{E}, \boldsymbol{P} \to 0$ as $|\tilde{z}| \to \infty$ (a so-called bright soliton condition), we derive a universal "EMB replication" between $\boldsymbol{E}$ and $\boldsymbol{P}$,

$$\boldsymbol{E}(\tilde{t}) = 4\pi\boldsymbol{P}(\tilde{t})M, \qquad M \equiv \beta^2/(1-\beta^2), \qquad (2)$$

with $\sqrt{M}$ being a relativistic EMB "momentum." It holds for *any* constitutive relationship between $\boldsymbol{P}$ and $\boldsymbol{E}$.

Consider first an EMB in a medium of quantum TLSs characterized by the dipole moment $\boldsymbol{d}$ and resonant frequency ω_0. Using the standard theory of TLS without relaxation and introducing normalized field $f \equiv 2\boldsymbol{d}\cdot\boldsymbol{E}/\hbar\omega_0 = 2\Omega_R/\omega_0$ (where $\Omega_R \equiv 2\boldsymbol{d}\cdot\boldsymbol{E}/\hbar$ is a Rabi frequency), polarization per atom p (thus, $\boldsymbol{P} = N\boldsymbol{d}p$, where N is the density of particles), population difference per atom η (we use the notations of Ref. [6], which considered high harmonics generation in a superdressed TLS), and time $\tau = (t - z/\beta c)\,\omega_0$, we obtain Bloch equations (without RWA) as

$$\dot{\eta} = -f\dot{p}, \qquad \ddot{p} + p = f\eta, \qquad (3)$$

where the overdot designates $\partial/\partial\tau$. [The invariant of Eq. (3) is a square of the Rabi sphere radius, $\eta^2 + p^2 +$

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$\dot{p}^2 = \text{inv} = 1$.] The EMB replication (2) is written now as $f = pQM$, where $Q \equiv 4\alpha N\lambda_0(d/e)^2$, e is the electron charge, $\lambda_0 = 2\pi c/\omega_0$, and $\alpha = e^2/\hbar c = 1/137$. Solving (3) together with the EMB replication for atoms being initially in equilibrium, $\eta \to 1$ at $|\tau| \to \infty$, we obtain a solitary, nonoscillating wave, consistent with [5]

$$f(\tau) = 2f_0 \operatorname{sech}[(\tau - C)f_0], \qquad \eta = 1 - fp/2, \quad (4)$$

where C is an integration constant, and $f_0 = (QM - 1)^{1/2}$ $[= (\Omega_R)_{\text{pk}}/\omega_0]$ is the half amplitude of the soliton directly related to its length ($\propto f_0^{-1}$) and velocity β; see Eq. (2). The minimal velocity, at $f_0 = 0$, is $\beta_{\min} = (1 + Q)^{-1/2}$. Shorter EMBs have a higher peak amplitude and move faster. The peak amplitude of an EMB with the length T at a half-peak field [i.e., $T = 2.63/(\Omega_R)_{\text{pk}}$] is $E_{\text{pk}} \approx 1.32\hbar/Td$.

EMB (4) holds also for any *amplifying* TLS with the *inversed* population difference, $\eta(|\tau| \to \infty) = -1$. In this case, $\eta = -1 - fp/2$, $f_0 = (-QM - 1)^{1/2}$, $M < 0$, and $\beta^2 > 1$ [7]. Larger EMBs here move slower, approaching the speed of light as their amplitude increases.

The Fourier spectrum of EMB, $S_f(\omega) \propto \operatorname{sech}[\pi\omega/2(\Omega_R)_{\text{pk}}]$, spreads from zero to the cutoff frequency, $\sim (\Omega_R)_{\text{pk}}$. Phase-portrait considerations show that with $f = p = 1 - \eta = 0$ at $|\tau| \to \infty$, the *nonoscillating* EMB (4) is the *only* soliton supported by the system. Therefore, RWA-based envelope 2π solitons are inconsistent with the exact soliton (4) (which indicates that higher-order RWA may render 2π solitons unstable at long distances). EMB (4) may be regarded as a "full Bloch" 2π soliton; by introducing phase (or area) $\phi_R(\tau) \equiv \int_{-\infty}^{\tau} f\,d\tau$, we get $\phi_R(\infty) = 2\pi$. (This points to a possible "full Bloch" area theorem.)

The solution (4) is valid within the limitations of our TLS model; in particular, the EMB must be shorter than all the atomic relaxation times, which still allows EMBs as long as $\sim 10^{-9}$ s. It is instructive to consider an example of Xe, with $\hbar\omega_0 \sim 8.44$eV, $d/e \sim 7$ Å (based on the superdressed TLS model for the high harmonics generation in Xe [6]). In this case, for $T = 12$ ps, $E_{\text{pk}} \sim 1$ kV/cm ($\ll \hbar\omega_0/d$). Longer pulses can still be considered within the TLS model with relaxation included. Of more interest, however, are the shortest and most intense pulses. As the EMB field approaches the atomic field ($\sim 10^8$–10^9 V/cm), the ionization potential dominates the EMB formation, limiting EMB length and amplitude. For such fields, the TLS model is invalid. We can, however, evaluate limitations on EMB within a classical 1D model of an atom, with a strongly nonlinear potential, $U(x)$, limited at $|x| \to \infty$, to allow for ionization; here x is the electron displacement. Bloch equations (3) are replaced then by a classical, normalized equation for the electron motion,

$$\ddot{p} + du(p)/dp = f(\tau). \quad (5)$$

Here $p \equiv x/x_0$, x_0 is an atomic characteristic size; $u(p) \equiv U(x)/U_0$, U_0 is a characteristic energy (e.g., the ionization potential); $\tau \equiv \tilde{t}\omega_0$; $\omega_0^2 = U_0/m_e x_0^2$, m_e is the mass of electron; and $f(\tau) \equiv eE(\tau)x_0/U_0$. The total polarization here is $P = Nxe$, and the EMB replication is still $f(\tau) = p(\tau)MQ$, with $Q = 4\pi N(ex_0)^2/U_0$. Note that for EMB, TLS Bloch equations (3) reduce to a simple Duffing equation for, e.g., p, $\ddot{p} - Ap + Bp^3 = 0$ [with $A = QM - 1$ and $B = (QM)^2/2$], which is isomorphic to Eq. (5) (with $f = pMQ$) for the simplest classical nonlinear potential, $u(p) = p^2/2 + ap^4$, $a = \text{const} > 0$. Thus, this classical potential can give rise to the same EMB solution, Eq. (4). For an arbitrary potential $u(p)$, the family of EMB solutions, $p(\tau)$, is found from Eq. (5) through the quadrature:

$$\int \{MQp^2 - 2[u(p) - u(0)]\}^{-1/2}\,dp = \pm\tau. \quad (6)$$

A "bright" solitary solution to (6) exists, however, only under a certain condition on nonlinearity. If, e.g., $u(p) = p^2/2 + ap^4$, the nonlinearity must be "positive," $a > 0$ [8]. In general, if u is a smooth, monotonically increasing function of p^2, the "bright" soliton exists only when $u(p) - u(0) > p^2\,du(0)/d(p^2)$ near $p = 0$. This requires the atomic potential to have sufficiently "hard walls," which holds for some model potentials [9(a)] [but not for such a "soft" potential as, e.g., $u = -(1 + p^2)^{-1/2}$ [9(b)]]. An example of potential with controllable "hardness" that allows for an analytic solution of (6) is $u(p) = p^2(1/2 + ap^2)/(1 + p^2)^2$, where $a = \text{const}$ [10]. To illustrate the limitations imposed by over-the-barrier ionization, consider first a "box" potential, $u(p) = 0$ for $|p| < 1$, and $u(p) = 1$ otherwise, in which case the solution of (6) is

$$p(\tau) = \exp(-|\tau|\sqrt{f_0}), \qquad f(\tau) = f_0 p(\tau) \qquad (f_0 \le 2), \quad (7)$$

and $M = f_0/Q$. (We presume here that an electron always starts its motion at $p = 0$.) Thus, the maximal field strength $E_{\max}$ and shortest EMB length $T_{\min}$ are

$$E_{\max} = 2U_0/ex_0, \qquad T_{\min} \approx (x_0/2c)\sqrt{m_e c^2/U_0}, \quad (8)$$

where $2x_0$ is a total box width. $E_{\max}$ is of the same nature as an atomic field, $E_{\text{at}} = E_{\max}/2$, i.e., the atom is ionized (in classical terms) by a pulse (7), if its amplitude exceeds $E_{\max}$; $T_{\min}$ is the time for such a field to pull an electron out of the potential well. With $U_0 = 20$ eV and $x_0 = 1$ Å, one has $E_{\max} \approx 4 \times 10^9$ V/cm, and $T_{\min} \sim 0.026$ fs. To make a connection to atoms with Coulomb long-range attraction, consider now a potential

$$u = 1 - (1 + 2bp^2 + p^4)^{-1/4}, \quad (9)$$

with $u - 1 \approx p^{-1}$ at $|p| \to \infty$. It has a single well and satisfies the hard-wall condition only when $0 \le b \le b_{\text{cr}} = \sqrt{2/5}$. For a given U_0 and atomic number Z we have $x_0 = r_e Z(m_e c^2/U_0)$, $\lambda_0 \equiv 2\pi c/\omega_0 = 2\pi r_e Z \times (m_e c^2/U_0)^{3/2}$, and $E_{\text{at}} = (2/5)^{1/4}U_0/ex_0$; here $r_e = e^2/m_e c^2$ is the classical electron radius. As an illustration, consider a limiting case, $b = 0$. Small-amplitude EMBs are governed again by a Duffing equation, $\ddot{p} - MQp + p^3 \approx 0$, with a solitary solution $p \approx p_0\sqrt{2}\,\mathrm{sech}(\tau p_0)$ (Fig. 1, curve 1). Here $p_0 = \sqrt{MQ}$, and, therefore, $\beta_{\min} = 0$, i.e., small EMBs here can move very slowly, the feature of any potential with $du(0)/d(p^2) = 0$. The EMB peak amplitude is $f_{\rm pk} = \sqrt{2}(MQ)^{3/2}$. Thus, as its amplitude increases, an EMB moves faster and shortens. However, at $p_{\rm pk} \approx (8/45)^{1/4} \approx 0.65$, $f_{\rm pk} \approx 0.122$, EMB length reaches minimum, $\tau_{A\,\min} \approx 5.3$ (at the half-peak amplitude, Fig. 1, curve 2) or $\tau_{I\,\min} \approx 2$ (at the half-peak intensity). Assuming $U_0 \approx 24$ eV and $Z = 2$, as in He, the shortest EMB length is $T_{I\,\min} \approx 2(r_e Z/c)(m_e c^2/U_0)^{3/2} \sim 10^{-16}$ s. If the EMB amplitude is further increased, EMB broadens and flattens (Fig. 1, curve 3). Finally, at a threshold amplitude, $p_{\rm pk} \approx 1.245$, $f_{\rm pk} \approx 0.42$, it becomes a shock (antishock) wave whose leading (trailing) edge is an autoionization (deionization) front (Fig. 1, curve 4). Its field rises (falls) as $\exp(\tau/\tau_{\rm ion})$, with $\tau_{\rm ion} \approx (MQ)^{-1/2} \approx 1.7$.

This shock wave is typical to any hard-wall ionization potential. Our preliminary results [10] indicate though that it may become unstable, producing a short precursor that travels as a pilot EMB at a faster speed ahead of the group of other longer and closely spaced EMBs merging into a dc field far behind it. In a more detailed picture of a shock wave, the classical over-the-barrier ionization near the threshold must be modified by quantum tunneling.

To demonstrate the EMB existence in both quantum and classical cases most rigorously, we used so far a "double-full" approach: full Maxwell equation (1) + full constitutive equations (3) or (5) (i.e., no RWA). Closer consideration shows, however, that (similarly to TLS in [5(b)]) of these two only constitutive equations are crucial, at least at low density, $Q \ll 1$, e.g., in gases, where typically $Q \sim 10^{-4}-10^{-1}$. In this case, the propagation velocity approaches the speed of light, $1 - \beta = O(Q)$. Writing the field as $\boldsymbol{E} = \boldsymbol{E}(\tilde{t}, \tilde{z})$, where, e.g., for the

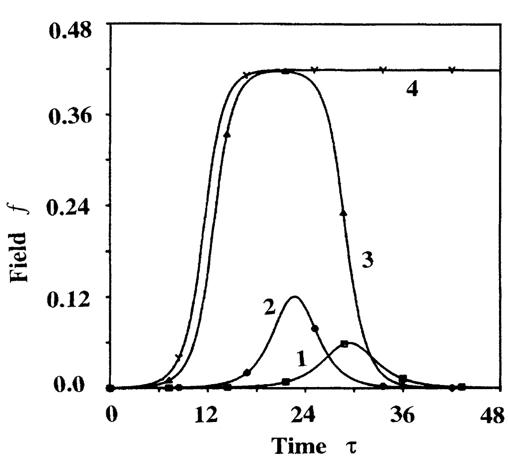

FIG. 1. Normalized field f vs time τ for steady state EMB (curves 1–3) and a shock wave (curve 4) due to ionization potential. Curves: 1—$MQ = 0.12$, 2—$MQ = 0.187$, 3—$MQ = (MQ)_{\rm ion} - 10^{-5}$; and 4—$MQ = (MQ)_{\rm ion} \approx 0.3403$.

wave propagating in the positive direction in z, $\tilde{t} = t - z/c$, $\tilde{z} = z$, and assuming that the field changes much slower in $\tilde{z}$ than in $\tilde{t}$, one can neglect the term $\partial^2 E/\partial \tilde{z}^2$ in the full Maxwell equation (1), reducing it to a first order wave equation $c\,\partial \boldsymbol{E}/\partial \tilde{z} + 2\pi\,\partial \boldsymbol{P}/\partial \tilde{t} = 0$. (The physical implication here is that nonlinear retroreflection is neglected.) The applicability of reduced Maxwell equation can be verified by, e.g., using it instead of Eq. (1) to obtain EMBs in either quantum or classical limits. In the transient propagation, it has also been verified by us in numerical simulations. To improve precision, we chose $\tilde{t} = t - z/\beta_{\min} c$; so that for, e.g., TLS, the reduced Maxwell equation is written as

$$-2c\,\partial \boldsymbol{E}/\partial \tilde{z}\,\beta_{\min} + Q\,\partial \boldsymbol{E}/\partial \tilde{t} = 4\pi\,\partial \boldsymbol{P}/\partial \tilde{t}\,. \qquad (10)$$

We found also that Eq. (10) can still be used even when Q is not small, if the field spectrum does not exceed ω_0.

One of the major issues is whether EMBs are feasible with currently available sources, e.g., half-cycle pulses or very intense short laser pulses, via a *transient propagation* process. In our computer simulations based on Eqs. (3) and (10), we choose Xe (see above) as TLS, and modeled a half-cycle pulse [2(a)] by a nonoscillating 400 fs long (at the half-peak amplitude) Gaussian pulse, with the amplitude $E_0 = 200$ kV/cm, Fig. 2(a), and 800 kV/cm (which is conceivably within the reach of current techniques, e.g., by using focusing); see Fig. 2(b). Figure 2 depicts the formation of a few distinct short EMBs out of a much longer pulse. [The amplitude of a *single* 400 fs long EMB (4) in Xe is only ~30 kV/cm.] As expected, a larger incident amplitude increases the number of EMBs. For $N \approx 10^{21}$ cm^{-3} (37 atm of Xe), the distance for the first EMB to appear at $E_0 = 200$ kV/cm is ~250 cm; however, it shortens dramatically (down to ~40 cm) at $E_0 = 800$ kV/cm. The shortest EMB in Fig. 2(a) is ~73 fs long at half-peak amplitude, and ~15 fs in Fig. 2(b) (with the fully developed EMBs being ~20%–30% shorter), and their amplitude is ~1.5 of the incident pulse. The TLS here is still not superdressed [6]: for $E_{\rm pk}$ in Fig. 2(b), $f_0 \sim 2 \times 10^{-2} \ll 1$, i.e., $(\Omega_R)_{\rm pk} \ll \omega_0$; thus the atom is far from the ionization.

We have also discovered that EMBs are remarkably stable against temporal or spatial changes of medium parameters. In particular, when the gas density N was varied by an order of magnitude along the path of propagation, the EMB profile and its length remained stable; only its velocity β was adjusting to a varying density N so that $M(\beta)Q(N) = \mathrm{inv}$. An EMB generated, e.g., in a gas jet, can slide into vacuum without distortion.

At this point, no mathematical proof exists that EMBs of "double-full" (Maxwell + constitutive) equations are real solitons in the sense of full integrability of these equations, and that, therefore, they are absolutely stable. Our numerical simulations for both TLS and nonlinear classical potentials show that small EMBs due to reduced Maxwell equation (10) are stable against both small and large (e.g., collision with another EMB) perturbations,

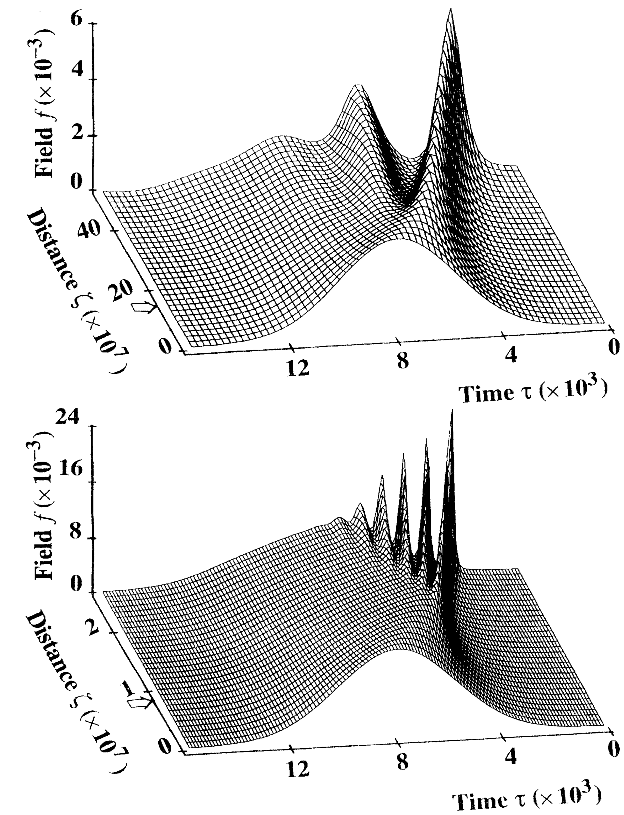

FIG. 2. EMBs originated in Xe by a 400 fs long Gaussian half-cycle pulse with different amplitudes, E_0: normalized field, $f = 2\mathbf{d} \cdot \mathbf{E}/\hbar\omega_0$, vs distance, $\zeta = 4\pi z \lambda_0^{-1}(1 + Q^{-1})^{-1/2}$, and time $\tau = \tilde{t}\omega_0$. (a) $E_0 = 200$ kV/cm, (b) $E_0 = 800$ kV/cm. Arrows indicate the first EMB formation.

which is consistent with results [5(b)] for TLS. Large EMBs approaching the ionization threshold may become unstable and break down into smaller EMBs.

Our preliminary numerical simulation for an *envelope* Gaussian pulse with *many* cycles, which modeled 20 fs, 10^{13} W/cm^2 pulse of a Ti:sapphire laser, showed that after a very short distance (less than 0.2 mm) each of the cycles gave rise to many subcycle, sub-fs EMBs propagating with different velocities, so that the picture rapidly became very complicated. (A cw input sinusoid shows, however, periodical in space field "revival" to the initial field shape, similarly to the TLS driven by a periodically modulated envelope [11].) Multi-EMB formation by an almost periodic incident field may be directly related to the high harmonic generation (HHG) in the TLS single-atom response [6,12], with EMBs having a spectrum reaching into far UV; it indicates that EMBs may play a significant role in the HHG phenomenon, including plateau formation.

In conclusion, we demonstrated feasibility of nonoscillating EM solitons, EM bubbles. They can exist in both quantum and classical limits, with their maximum amplitude and minimum length limited by the atomic ionization. We showed that femtosecond and sub-fs EMBs can be generated by available sources of EM radiation.

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[7] Since a TLS with $\eta_\infty = -1$ is a *nonequilibrium* system storing energy of the external pumping, β here is not the speed of energy propagation, which explains $\beta^2 > 1$.

[8] If nonlinearity is negative, $a < 0$, one can expect formation of "dark" EMB (a solitary "hole" propagating on a cw field background): $f(\tau) \propto \tanh(\tau f_0)$, $f_0 = \text{const}$.

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