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SUBFEMTOSECOND HIGH-INTENSITY UNIPOLAR ELECTROMAGNETIC SOLITONS AND SHOCK WAVES

A. E. KAPLAN* and P. L. SHKOLNIKOV
Department of Electrical and Computer Engineering,
The Johns Hopkins University, Baltimore, MD 21218, USA

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We show that atomic gasses can support solitary pulses of unipolar, non-oscillating EM-field ("EM-bubbles") of up to the atomic amplitude, with their length ranging from $\sim 10^{-9}$ s to $\sim 10^{-16}$ s, which propagate without dispersion and are stable and insensitive to the change of gas density. We found the condition on the atomic potential necessary for formation of EM-bubble. Atomic gasses can also support an EM shock which is a precursor of a *cw* ionizing field.

All the experimentally observed time-domain solitons in nonlinear optics, are so-called envelope solitons, i.e. quasi-harmonic oscillations modulated by an envelope much longer than a single cycle of the carrier wave, with their spectral width substantially smaller than their carrier frequency. Typical examples are optical-fiber solitons[1] due to Kerr-nonlinearity, described by a nonlinear Schrödinger equation,[2] and 2-π solitons due to Rabbi flipflop in two-level systems (TLS),[3] described by Maxwell–Bloch or sin-Gordon equations. All these descriptions use slow-varying envelope approximations in both propagation (by reducing Maxwell equations to a parabolic partial differential equation) and in material response (rotating wave approximation (RWA) in constitutive equations). Many applications, however, in particular the study of atomic physics by means of photoionization, call for short and intense EM-pulses of *non-oscillating* nature, with their spectrum spread from zero to some cutoff frequency. Atomic ionization with almost unipolar 'half-cycle pulses' has been of substantial interest recently (see e.g. Ref. 4). Currently available pulses generated in semiconductor structures are ~ 400 fs long with the peak field of $\sim 10^5$ V/cm.

Shorter (10^{-14} s – 10^{-16} s) and more intense (up to 10^9 V/cm) unipolar pulses might be of great interest for the host of applications. They can be used for a 'global' spectroscopic technique based on a shock-like excitation across the entire atomic spectrum (to the extent similar to passing atoms through a foil), including

*Corresponding author.

the normally prohibited transitions. The ionization by a pulse shorter than the orbital period may bridge a gap between conventional photonization and collisional ionization by a particle;[5] it can also provide an exciting opportunity to pick up an electron at a certain point of its orbit, provided its phase has been prepared before the event. Time-domain spectroscopy of dielectrics and semiconductors with these pulses may expand this method from presently available THz domain (see e.g. Ref. 7) to optical frequencies. The need in such pulses is obvious in the time-resolved spectroscopy of transient chemical processes occurring on a femotosecond time scale, e.g. dissociation and autoionization (see e.g. Ref. 8), and especially for quantum control of chemical transformations (see e.g. Ref. 9). One can easily envision their applications for probing high-density plasmas in Tokamaks, new tests of the speed of light, imaging of molecules and atoms at the surfaces, pulse-dispersion density measurement of very rarefied gasses, etc. Another potential application is an order of magnitude frequency upconversion due to large Doppler shift of a counterpropagating coherent light backscattered by EMB.

Super-short pulses can be obtained only by using strongly nonlinear processes.[10] Especially significant are non-oscillating *solitary* waves able to propagate over substantial distances with unchanged shape and length. The exact soliton-like solutions for the nonlinear propagation of unipolar pulses in the strongly-driven TLS, described by full Maxwell + full Bloch equations, were found quite a while ago.[11] The solution has a familiar, *sech* profile, with its duration and velocity related to its amplitude.

In our most recent research[12] we showed that such solitons are not only feasible, but also a natural process for many nonlinear system, both quantum and classical. Their length may range from a small fraction of the cycle length of the resonance that supports them, to much longer than that cycle, depending on their intensity. We will call them EM-bubbles (EMB) to stress their non-envelope nature. We show that with the light intensities available now, these EMBs can be as short as 10^{-16} s. We demonstrated[12] that photoionization imposes an upper limit on the EMB amplitude and a lower limit on its length; after an EMB reaches its shortest length at some peak amplitude, the further increase of the amplitude results in EMB broadening. At some threshold amplitude, the EMB degenerates into a shock wave of ionization.

For a plane EM-wave propagating along the z-axis, the Maxwell equation for the electric field $\mathbf{E}$ is

$$\frac{c^2\partial^2\mathbf{E}}{\partial z^2} - \frac{\partial^2\mathbf{E}}{\partial t^2} = \frac{4\pi\partial^2\mathbf{P}}{\partial t^2}\,. \tag{1}$$

where $\mathbf{P}$ is polarization. Assuming that an EMB propagates with a constant velocity, βc, introducing retarded variables, $\tilde{t} \equiv t - z/\beta c$ and $\tilde{z} = z$, imposing a steady state condition, $\partial\mathbf{E}/\partial\tilde{z} = \partial\mathbf{P}/\partial\tilde{z} = 0$, we readily obtain the relationship:

$$\frac{\partial^2\mathbf{E}}{\partial\tilde{t}^2} = 4\pi M \cdot \frac{\partial^2\mathbf{P}}{\partial\tilde{t}^2}\,;$$

where

$$M \equiv \frac{\beta^2}{(1-\beta^2)} = \text{const}\,,$$

with $\sqrt{M}$ being a normalized relativistic 'momentum' of an EMB. Stipulating now that the EMB carries finite energy, i.e. $\mathbf{E},\ \mathbf{P} \to 0$ as $|\tilde{z}| \to \infty$ (a so-called bright soliton condition), we obtain a universal 'replication' relationship between $\mathbf{E}$ and $\mathbf{P}$[12]:

$$\mathbf{E}(\tilde{t}) = 4\pi M \mathbf{P}(\tilde{t})\,. \tag{2}$$

Equation (2) is valid for *any* constitutive relationship between $\mathbf{P}$ and $\mathbf{E}$.

Consider first the pulse propagation in a medium with quantum TLS characterized by the dipole momentum, $\mathbf{d}$, and resonant frequency, ω_0. Using the standard theory of TLS without relaxation, and introducing normalized variables: field

$$f \equiv \frac{2\mathbf{dE}}{\hbar\omega_0} = \frac{2\Omega_R}{\omega_0}\,,$$

(where $\Omega_R \equiv \mathbf{dE}/\hbar$ is Rabi frequency), polarization per atom, $p = \rho_{12} + \rho_{21}$, (with the total polarization being $\mathbf{P} = N\mathbf{d}p$ where N is the density of particles), population difference per atom, $\eta = \rho_{11} - \rho_{22}$, where ρ_{jk} $(j,k = 1,2)$ are density matrix elements of the system (with $\rho_{11} + \rho_{22} = 1$ and $\rho_{12} = \rho_{21}^*$), (we use the same notation here as in Ref. 13), and $\tau = (t - z/\beta c)\omega_0$, we obtain Bloch equations (without RWA or any other approximation) as[13]

$$\dot{\eta} = -f\dot{p}; \quad \ddot{p} + p = f\eta\,, \tag{3}$$

where the 'dot' designates $\partial/\partial t$. The first integral of Eq. (3) is the square of the Rabi sphere radius,

$$\eta^2 + p^2 + \dot{p}^2 = \text{const} = 1\,.$$

The replication (2) is written now as $f = pQM$, where

$$Q \equiv 4\alpha N \lambda_0 (d/e)^2\,;$$

e is the electron charge, $\lambda_0 = 2\pi c/\omega_0$ and $\alpha = e^2/\hbar c = 1/137$ is the fine structure constant. Solving (3) together with the replication condition for atoms being initially in equilibrium, $\eta \to 1$ at $|\tau| \to \infty$, we obtain a solitary, non-oscillating wave, consistent with Ref. 11.

$$f(\tau) = 2f_0 \text{sech}[(\tau - C)f_0]; \quad \eta = 1 - \frac{fp}{2}\,; \tag{4}$$

where C is an integration constant, and $f_0 = \sqrt{QM - 1}$ is the amplitude of the soliton directly related to its length, $\tau_0 = f_0^{-1}$, and to its velocity, $\beta = \sqrt{M/(1+M)}$. The minimal (critical) velocity $\beta_{cr} = (1+Q)^{-1/2}$ corresponds to $f_0 = 0$.

An EMB has a Fourier spectrum

$$S_f(\omega) \propto \operatorname{sech}\left(\frac{\pi\omega}{2f_0\omega_0}\right),$$

spreading from $\omega = 0$ to the cutoff frequency $\sim f_0\omega_0$. Phase-portrait considerations show that with $f = p = 1 - \eta = 0$ at $|\tau| \to \infty$, the *non-oscillating* exact solution (4) is the *only* type of solitary wave supported by the system. Therefore, surprisingly, RWA-based SIT envelope 2π-solitons are not consistent with the exact solution (4). This may indicate that higher order RWA must render SIT solitons unstable at long enough distances.

The solution (4) is valid within the limitations of our TLS model. In particular, the EMB duration, $t_0 = \tau_0/\omega_0$, must be shorter that all the atomic relaxation times, which still allows for EMBs as long as $\sim 10^{-9}$ s, with longer EMBs having lower peak amplitude,

$$E_{pk} = \frac{\hbar\omega_0 f_0}{2d} = \frac{\hbar}{2dt_0},$$

and moving slower,

$$\beta = \left[\frac{1+Q}{(1+f_0^2)}\right]^{-1/2}.$$

It is instructive to consider an example of Xe, with $\hbar\omega_0 \sim 8.44$ eV, dipole size, $d/e \sim 7$ Å (based on the 'super-dressed TLS' data for high harmonics generation in Xe[13]), and $N \sim 10^{19}$ cm^{-3}. In this case, $Q \sim 10^{-2}$, and for a 10 ps long EMB, $E_{\rm pk} \simeq 10^3$ V/cm. Longer pulses can still be considered within the TLS model with relaxation included. However, of particular interest are the shortest and most intense pulses. As the EMB field approaches the atomic field ($\sim 10^8 - 10^9$ V/cm), the ionization potential dominates the EMB formation, limiting EMB length and amplitude. For such fields, TLS model is invalid. We can, however, evaluate limitations on EMB within a classical 1-D model of an atom, with a strongly nonlinear potential, $U(x)$, limited at $|x| \to \infty$, to allow for ionization; here x is the electron displacement. Then Bloch equations (3) are replaced by a classical normalized equation for the electron motion:

$$\ddot{\xi} + \frac{du(\xi)}{d\xi} = f(\tau). \qquad (5)$$

Here $\xi \equiv x/x_0$, x_0 is an atomic characteristic size; $u(\xi) \equiv U(x)/U_0$, U_0 is a characteristic energy (e.g., the ionization potential)[14]; $\tau \equiv \tilde{t}\omega_0$;

$$\omega_0^2 = \frac{U_0}{m_e x_0^2};$$

where m_e is the mass of electron; and $f(\tau) \equiv eE(\tau)x_0/U_0$ is a normalized driving field. The total polarization here is $P = Nxe$, and the replication formula is now $f(\tau) = \xi(\tau)MQ$, with $Q = 4\pi N(ex_0)^2/U_0$. Note that for EMB, TLS Bloch equations (3) reduce to a simple Duffing equation, for, e.g. p,

$$\ddot{p} - Ap + Bp^3 = 0\,,$$

[with $A = QM - 1$ and $B = (QM)^2/2$], which is isomorphic to Eq. (5) (with $f = \xi MQ$) for the simplest classical nonlinear potential,

$$u(\xi) = \frac{\xi^2}{2} + a\xi^4\,,$$

with $a = \text{const} > 0$. Thus, this classical potential can give rise to the same EMB solution, Eq. (4). For an arbitrary potential $u(\xi)$, the family of EMB solutions, $\xi(\tau)$, is found from Eq. (5) through the quadrature[12]:

$$\int \frac{d\xi}{\sqrt{MQ\xi^2 - 2[u(\xi) - u(0)]}} = \pm\tau\,. \tag{6}$$

A 'bright' solitary solution to Eq. (6), exists, however, only for a particular class of nonlinearities. For example, if $u(\xi) = \xi^2/2 + a\xi^4$, the nonlinearity must be 'positive', $a > 0$ (Ref. 15). More generally, if $u(\xi)$ is a smooth function, monotonically increasing with ξ^2, and $u(\xi) = u(-\xi)$, the 'bright' solitary solution exists only if

$$u(\xi) - u(0) > \frac{\xi^2 du(0)}{d(\xi^2)}$$

at some $\xi \neq 0$. This condition requires the atomic potential to have sufficiently 'flat bottom' or 'hard walls'. For example, a model potential, $u = -(1+\xi^2)^{-1/2}$ frequently used in the theory of highly nonlinear optical processes[16] *cannot* give rise to EMBs. However, many model potentials[17] satisfy the 'hard-wall' condition. An example of a model potential that allows for an explicit analytic solution of Eq. (6) is[12,18]

$$u(\xi) = \frac{\xi^2(\xi^2 + b/2)}{(I + \xi^2)^2}$$

where $b = \text{const} > 0$. To roughly estimate the shortest possible time and the largest EMB amplitude due to over-the-barrier ionization, consider first a classical 'box' potential, $u(\xi) = 0$ for $|\xi| < 1$, and $u(\xi) = 1$ otherwise, in which case the EMB field is

$$f(\tau) = f_0 \exp(-|\tau - C|\sqrt{f_0})\,, \quad \text{with} \quad f_0 \leq 2\,, \tag{7}$$

and $M = f_0/Q$. (We presume here that an electron always start its motion at $\xi = 0$.) Thus, the maximal field strength, $E_{\max}$, and shortest EMB length, $t_{\min}$, are

$$E_{\max} = \frac{U_0}{ex_0}; \quad t_{\min} = \left(\frac{x_0}{c}\right)\sqrt{m_e c^2/2U_0}\,; \tag{8}$$

where U_0 is an ionization limit, and $2x_0$ is the total box width. E_{max} is of the same nature as an atomic field, $E_{at} = E_{max}/2$, i.e. the atom is ionized (in classical terms) by a pulse of a certain shape [here, Eq. (7)], if its peak amplitude exceeds E_{max}; t_{min} is the time required for such a field to pull an electron out of the potential well. (With $U_0 = 20$ eV and $x_0 = 1$ Å, this results in $E_{max} \approx 2 \cdot 10^9$ V/cm, and $t_{min} \sim 0.4 \cdot 10^{-16}$ s). To make a connection to atoms with Coulomb long-range attraction, consider now a potential

$$u = 1 - (1 + 2b\xi^2 + \xi^4)^{-1/4} \tag{9}$$

with $u - 1 \approx \xi^{-1}$, at $|\xi| \to \infty$. It has a single well and satisfies the flat-bottom condition only when $0 \leq b \leq b_{cr} = \sqrt{2/5}$; if $b > b_{cr}$ this potential cannot support EMBs. For a given U_0 and atomic number, Z, we have

$$x_0 = r_e Z \cdot (m_e c^2/U_0); \quad \lambda_0 \equiv 2\pi c/\omega_0 = 2\pi r_e Z (m_e c^2/U_0)^{3/2},$$
$$\text{and} \quad E_{at} = (2/5)^{1/4} U_0/ex_0\,;$$

here $r_e = e^2/m_e c^2$ is the classical electron radius. As an illustration, consider a limiting case with $b = 0$. Small-amplitude EMBs are governed again by a Duffing equation, $\ddot{\xi} - MQ\xi + \xi^3 \approx 0$, with its solitary solution being $\xi = \xi_0\sqrt{2}\text{sech}(\tau\xi_0)$ (Fig. 1, curve 1). Here $\xi_0 = \sqrt{MQ}$, and therefore, $\beta_{cr} = 0$, i.e. small-amplitude EMBs here can move very slowly, a typical feature of any potential with $du(0)/d(\xi^2) = 0$. The EMB peak amplitude is $f_{pk} = \sqrt{2(MQ)^3}$. Thus, as its amplitude increases, a EMB moves faster, and shortens. However, at $\xi_{pk} \approx (8/45)^{1/4} = 0.65$, $f_{pk} = 0.122$, EMB length (at the half-peak amplitude) reaches minimum, $\tau_{A\,min} \approx 5.3$ (at the half-peak amplitude, Fig. 1, curve 2) or $\tau_{I\,min} \approx 2$ (at the half-peak intensity). Assuming $U_0 \approx 24$ eV and $Z = 2$, as in He, one obtains the shortest EMB length:

$$t_{I\,min} = 2(r_e Z/c)(m_e c^2/U_0)^{3/2} \sim 10^{-16}\ \text{s}\,.$$

Significantly shorter EMBs can be attained with ionized atoms, e.g. ion beams, which may have ionization potential, U_0, orders of magnitude larger. As the field amplitude continues to rise, EMB begins to broaden, becoming a flat-top pulse (Fig. 1, curve 3). Finally, at a threshold amplitude, $\xi_{pk} \approx 1.245$, $f_{pk} \approx 0.42$, it becomes a shock (anti-shock) wave whose single leading (trailing) edge is a front of an ionizing (deionizing) *cw* field. (Fig. 1, curve 4). The amplitude front rises (falls) as $\exp(\tau/\tau_{ion})$, with

$$\tau_{ion} \approx (MQ)^{-1/12} \approx 1.7\,.$$

This shock wave is typical to any potential with ionization, satisfying hard-wall condition. Our preliminary results[18] indicate though that a single-front shock wave becomes unstable, producing a short precursor that travels as a pilot EMB at a faster speed ahead of the group of other, longer and closely spaced EMBs, which merge into a *cw* field far behind the precursor. This pattern persists if one accounts

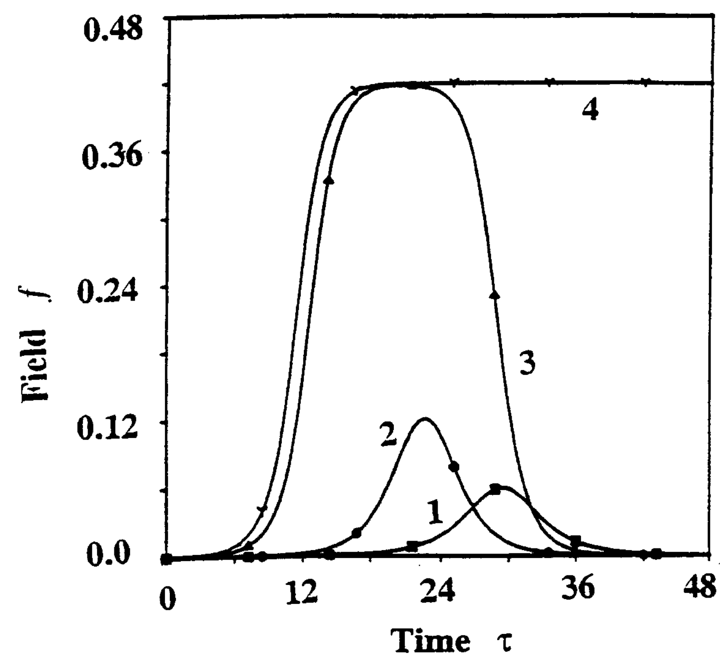


Fig. 1. Normalized field amplitude, f vs time τ, for steady-state EMB (curves 1–3) and a shock wave (curve 4) due to ionization potential. Curves: (1) $MQ = 0.12$ (2) $MQ = 0.187$ (3) $MQ = (MQ)_{\text{ion}} - 10^{-5}$ and (4) $MQ = (MQ)_{\text{ion}} \approx 0.3403$.

for the plasma due to ionization behind the pilot group of EMBs. Low-density free electrons are generated here by the quantum tunneling when the EMB amplitude approaches atomic field.

To demonstrate the existence of EMBs (in both quantum and classical cases) most rigorously, we have used so far a 'double-full' approach: full Maxwell equation (1) + full constitutive Eqs. (3) or (5) (i.e. without RWA). Closer consideration shows, however, that (similar to Ref. 11b where this was shown for a TLS) out of these two theoretical components only the latter one is crucial, at least for low density, $Q \ll 1$, e.g. in gases, where typically, $Q \sim 10^{-4} - 10^{-1}$. In this case, the propagation velocity approaches the speed of light, $1 - \beta = 0(Q)$. Writing the field as $\mathbf{E} = \mathbf{E}(\tilde{t}, \tilde{z})$, where, e.g. for the wave propagating in the positive direction in z, $\tilde{t} = t - z/c$, $\tilde{z} = z$, and assuming that the field changes much slower in $\tilde{z}$ than in $\tilde{t}$, one can neglect the term $\partial^2 E/\partial \tilde{z}^2$ in the full Maxwell Eq. (1) reducing it to a first order wave equation

$$\frac{c\partial \mathbf{E}}{\partial \tilde{z}} + \frac{2\pi \partial \mathbf{P}}{\partial \tilde{t}} = 0\,. \tag{10}$$

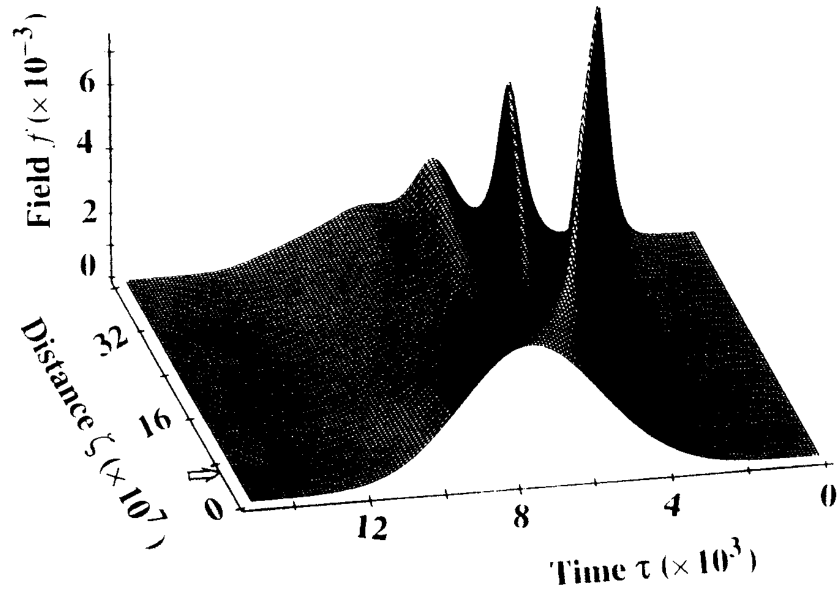
(The physical implication here is that nonlinear retroreflection is neglected.) The validity of the reduced Maxwell equation can be verified by e.g. using it instead of Eq. (1) to obtain EMBs in either quantum and classical limits. In the transient propagation, it has also been verified by us in numerical simulations. To improve precision, we choose $\tilde{t} = t - z/\beta_{\text{cr}}c$; so that for e.g. TLS, the reduced Maxwell equation is written as

$$\frac{-2c\partial\mathbf{E}}{\partial\tilde{z}\beta_{cr}} + \frac{Q\partial\mathbf{E}}{\partial\tilde{t}} = 4\pi\,\frac{\partial\mathbf{P}}{\partial\tilde{t}}\,. \tag{11}$$

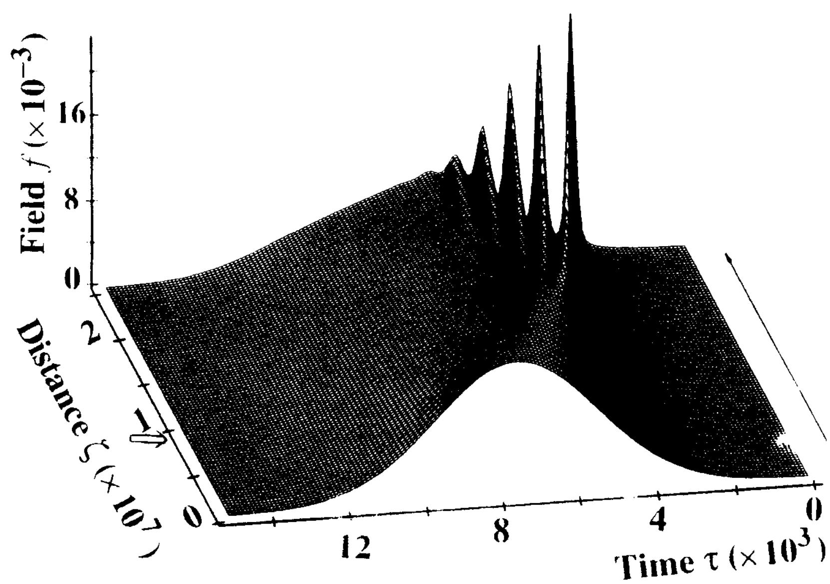
We also found that Eq. (11) can still be used even if Q is not small, if the field spectrum does not exceed ω_0.

As to the feasibility of experimental observation of EMBs, one of the major, yet never addressed even for TLS, issues is whether EMBs can be excited using currently available sources, e.g. half-cycle pulses or very intense short laser pulses with relatively few cycles, via transient propagation process. In our computer simulations we found that distinct individual EMBs can be obtained with a half-cycle pulse. The EMBs formation based on the solution of Eqs. (3) and (11), is shown in Fig. 2, which depicts nonlinear propagation of such a pulse with different amplitudes, E_0. We modeled a half-cycle pulse by a nonoscillating 400 fs Gaussian pulse, with the peak amplitudes of $E_0 \sim 250$ kV/cm somewhat larger than the one reported in Ref. 4, Fig. 2a, and 800 kV/cm, Fig. 2b (larger than that reported in Ref. 4, but conceivably within the reach of current experimental techniques). The parameters of the TLS were chosen to correspond to Xe (Ref. 13), see above, with $N \approx 10^{21}$ cm^{-3} (37 atm of Xe), and thus $Q \sim 2$. The propagation distance for the first EMB to appear is ~ 2.5 m for $E_0 \sim 250$ kV/cm, and ~ 40 cm for $E_0 \sim 800$ kV/cm. The narrowest EMB in Fig. 2 is ≈ 10 fs wide (on half-intensity level), with its amplitude being 50 % larger than the original Gaussian pulse. Figure 2 shows the formation of few very short EMBs out of much broader initial pulse. (Formation of more than one EMB in general is consistent with the existence of multi-soliton solutions.[11b] Note that in this simulation, the field is still below the super-dressed regime of TLS, $f \ll 1$ ($\Omega_R \ll \omega_0$), and therefore far from the ionization limit. As one may expect, the increase of the incident amplitude results in the increase in the number of EMBs. A critical amplitude to observe only one EMB for the example considered here, is as low as ~ 30 kV/cm.

The possibility of the EMB formation in *each* laser cycle increases tremendously (although the ensuing picture becomes much more complicated due to the multiple EMB interactions) when the regular laser radiation with many cycles in the envelope is used instead of half-cycle pulses. Indeed, since the laser cycle is much shorter (e.g., the cycle duration for the radiation with $\lambda = 0.9$ μn is 3 fs), and the laser intensities of, e.g. 10^{14} W/cm^2 (which correspond to the field $\sim 2.7 \times 10^8$ V/cm), are readily available now, the distance at which the EMB formation occurs, becomes shorter than 1 mm, and the bubble length becomes an order of magnitude shorter than the optical cycle. The very beginning of braking of an *individual* laser cycle into much shorter EMBs is shown in Fig. 3, for the relatively low laser intensity 6×10^{11} W/cm^2. For the case of Xe with the density $Q = 0.66 \times 10^{-2}$ (0.1 atm), the total stretch of propagation along the z-axis shown in Fig. 3, is 0.3 mm, and the formation of the first EMB can be seen at the short distance, ~ 0.1 mm. With more intense laser light, our preliminary calculations with 10 fs-long, 10^{12} W/cm^2 pulse of a Ti:Spph laser, indicate that after a very short distance (less than 0.1 mm) the central (most



(a)



(b)

Fig. 2. EMBs originating by nonlinear propagation of an initially Gaussian non-oscillating pulse for different amplitudes, E_0, of the incident pulse. The dimensionless field is $f = 2dE/\hbar\omega_0$, propagation distance is $\zeta = 4\pi z\sqrt{Q/(1+Q)}/\lambda_0$; and time is $\tau = t\omega_0$. A thin arrow on Fig. 2b indicates the direction of propagation and a thick arrow indicates the first EMB formation. (a) $E_0 = 250$ kV/cm (b) $E_0 = 800$ kV/cm.

intense) oscillations of the pulse begin to split into many EMBs, which propagate with different velocities, so that the picture rapidly becomes very complicated. (It is worth noting that there is a distinct possibility that the phenomenon of the very high harmonics generation[19] in noble gasses might be to a substantial degree attributed[18] to the multiple EMB formation, which would help to explain many major features of the HHG phenomenon, such as its puzzling insensitivity to the phase mismatch at different high harmonics, broadening and shift of harmonic spectra, etc.) A *cw* input sinusoid, similar to that depicted in Fig. 3, demonstrates spatially-periodical 'field revival' to the initial field configuration. This revival effect is reminiscent of the time-spatial revival of the envelope of a strong resonant periodically-modulated field propagating in the gas of two-level atoms.[20]

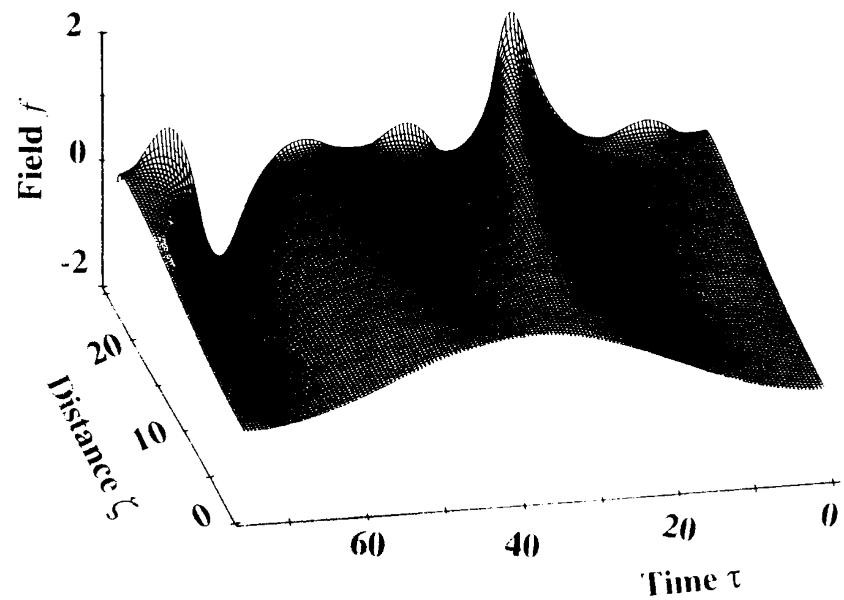


Fig. 3. EMBs originated by an initially sinusoidal laser oscillation; an individual *single* laser cycle is seen to start breaking into subcycle pulses about 10 times shorter than the original cycle.

In a related simulation, we have discovered that the EMBs are remarkably stable upon temporal or spatial changes of medium parameters. In particular, when the gas density, N, along the path of propagation was changed by an order of magnitude, the EMB profile and its length remained stable; only its velocity, β, was adjusting to a varying density. This suggests an experimental configuration with the incident pulse injected into a gas jet and resulting EMB exiting from the jet into a vacuum without any changes. At this point, no mathematical proof exists that in general, 'double-full' formulation, EMBs are real solitons in the sense of full integrability of the full Maxwell + full constitutive equations, and that, therefore, EMBs are absolutely stable. Our numerical simulation for both TLS and nonlinear classical potentials, show that small EMB due to reduced Maxwell equation (11) are stable against both small and large (e.g. collision with another EMB) perturbations, which is consistent with the results[12] for TLS. Large EMBs approaching the ionization threshold, may become unstable and breakdown into smaller EMBs.

In conclusion, we demonstrated feasibility of non-oscillating EM solitary waves, EM-bubbles. They can exist in both quantum and classical limits, with their maximum amplitude and minimum length limited by the atomic ionization. We demonstrated that 10 – 0.1 fs EMBs can be generated by the available sources of EM radiation.

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