Course ECE 520.482, Introduction to Lasers

Instructor: Prof. Alex Kaplan

Problem : flat+spherical mirrors resonator

A resonator is comprised of two mirrors: one flat, and another -- spherical, with the radious of curvature R; its axis of symmetry is normal to the flat morrir. The spacing between the mirrors is L; assume refractive index ~ 1. Find the size (radius) of the laser beam with a wavelength λ at both the mirrors, using diffraction theory of Gaussian beams found in the class notes
http://striky.ece.jhu.edu/˜sasha/COURSES/Gauss.diffr.pdf

Solution

In the stationary mode, the radius of curvature of a Gaussian beam has to coincide with that of the spherical mirror of the resonator. On the other hand, the waist of the beam (i. e. point of its flat phase front) should coincide with a flat mirror. From eq (4.29) we have for the radius of curvature, R of a Gaussina beam at the distance L from the waist

$$R = l_D\left(\frac{L}{l_D} + \frac{l_D}{L}\right) = L + \frac{l_D^2}{L}\ ; \quad \text{with} \quad l_D = ka^2\ ; \quad k = \frac{2\pi}{\lambda} \tag{1}$$

where a is the beam radius at the waist, i. e. at the flat mirror. From here we calculate the radius a to be

$$a = [L(R-L)]^{1/4}\sqrt{\lambda/2\pi} \tag{2}$$

i. e. the solution exists only if $R > L$, which satisfies the condition of ray stability in such a resonator.

The size (radius), ρ, of the Gaussian beam at the spherical mirror is determined from eq (4.29) in the above notes, as

$$\rho = a\sqrt{1+(L/l_d)^2} = \sqrt{a^2+(L/ka)^2} = \sqrt{\frac{R\lambda}{2\pi(R-L)}} \tag{3}$$