HW1 Solution

#1. Plot real and imaginary parts of susceptibility (χ), dielectric constant (ε_r), refractive index (n), and absorption coefficient as a function of wavelength. (30 points)

(Ref : Section 3C of the Chapter 3 notes)

Susceptibility is defined by:

$$\chi(\omega)=\frac{\omega_p^2}{\omega_0^2-\omega^2-j\omega\gamma}=\chi'(\omega)+\chi''(\omega) \tag{2.1}$$

Now $\omega=\frac{2\pi c}{\lambda}$, so we can write the equation as a function of wavelength:

$$\mathrm{Re}(\chi(\omega))=\chi'(\omega)=\frac{\omega_p^2(\omega_0^2-\omega^2)}{(\omega_0^2-\omega^2)^2+(\omega\gamma)^2} \text{ or } \chi'(\lambda)=\frac{\omega_p^2\lambda^2(\lambda^2\omega_0^2-(2\pi c)^2)}{(\lambda^2\omega_0^2-(2\pi c)^2)^2+(2\pi c\lambda\gamma)^2} \tag{2.2}$$

$$\mathrm{Im}(\chi(\omega))=\chi''(\omega)=\frac{\omega_p^2(\omega\gamma)}{(\omega_0^2-\omega^2)^2+(\omega\gamma)^2} \text{ or } \chi''(\lambda)=\frac{\omega_p^2\lambda^3(2\pi c\gamma)}{(\lambda^2\omega_0^2-(2\pi c)^2)^2+(2\pi c\lambda\gamma)^2} \tag{2.3}$$

where $\omega_p=\left(\frac{Ne^2}{m\varepsilon_0}\right)^{\frac{1}{2}}$ is known as the plasma frequency.

Dielectric constant is related to susceptibility by the following relation: $\varepsilon_r=1+\chi$. Hence,

$$\mathrm{Re}(\varepsilon_r(\lambda))=1+\frac{\omega_p^2\lambda^2(\lambda^2\omega_0^2-(2\pi c)^2)}{(\lambda^2\omega_0^2-(2\pi c)^2)^2+(2\pi c\lambda\gamma)^2} \text{ and} \tag{2.4}$$

$$\mathrm{Im}(\varepsilon_r(\lambda))=\frac{\omega_p^2\lambda^3(2\pi c\gamma)}{(\lambda^2\omega_0^2-(2\pi c)^2)^2+(2\pi c\lambda\gamma)^2}=\chi''(\lambda) \tag{2.5}$$

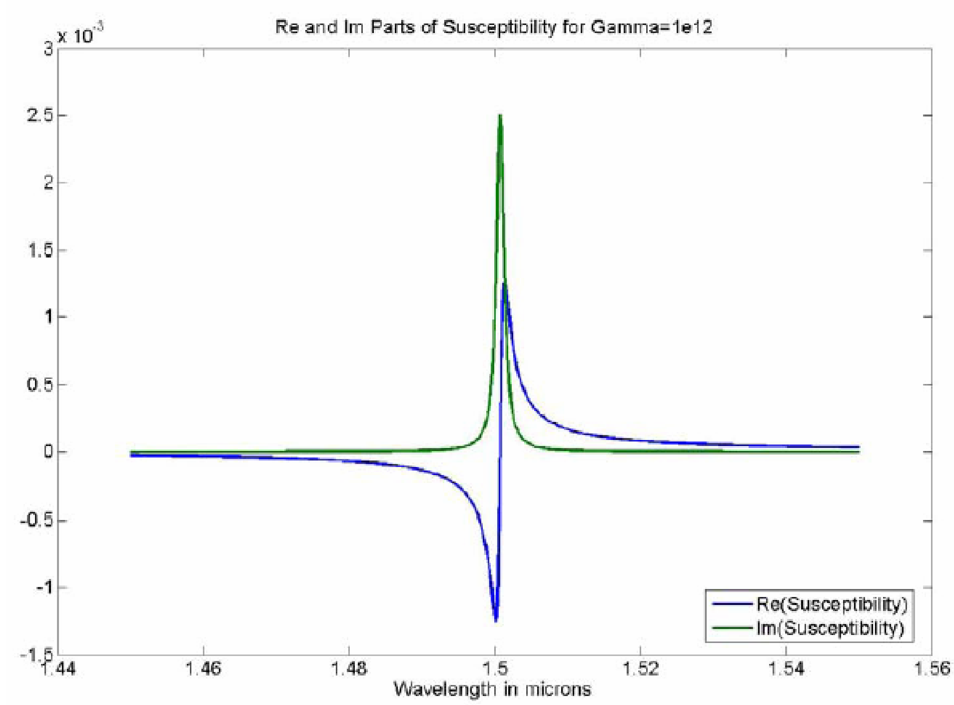
Refractive index is defined as: $n=\sqrt{\varepsilon_r(\lambda)}=n'(\lambda)+j\kappa(\lambda)$. Now we can make an approximation that we are dealing with low absorption material ($\chi' \gg \chi''$), and hence,

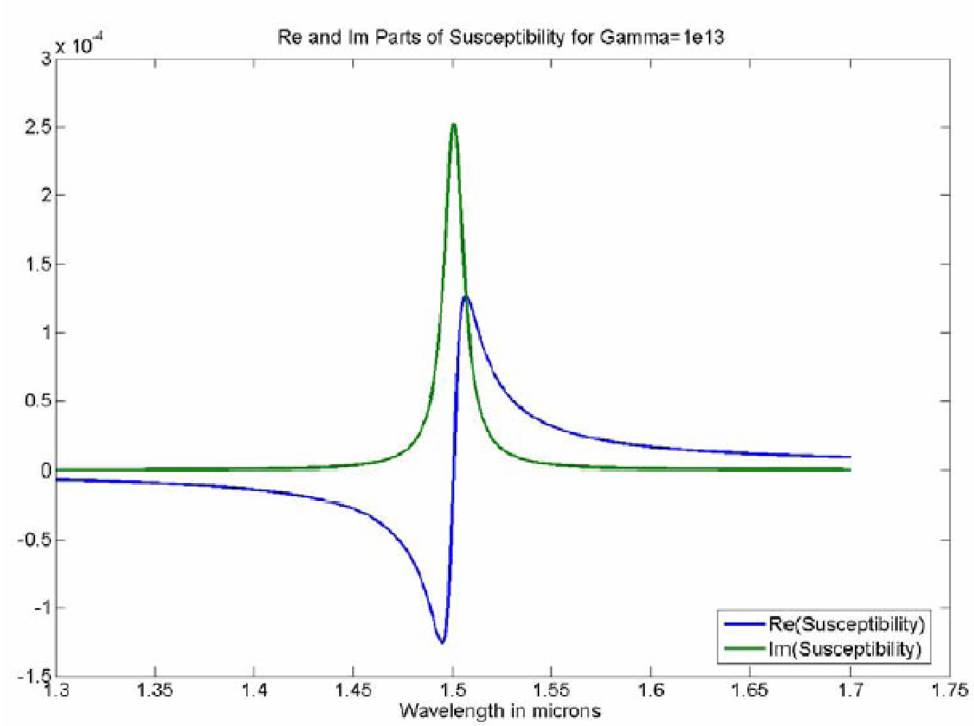
$$n'(\lambda)=\sqrt{1+\chi'(\lambda)}, \text{ and } \kappa(\lambda)=\frac{\chi''(\lambda)}{2n'(\lambda)}=\frac{\chi''(\lambda)}{2\sqrt{1+\chi'(\lambda)}} \tag{2.6}$$

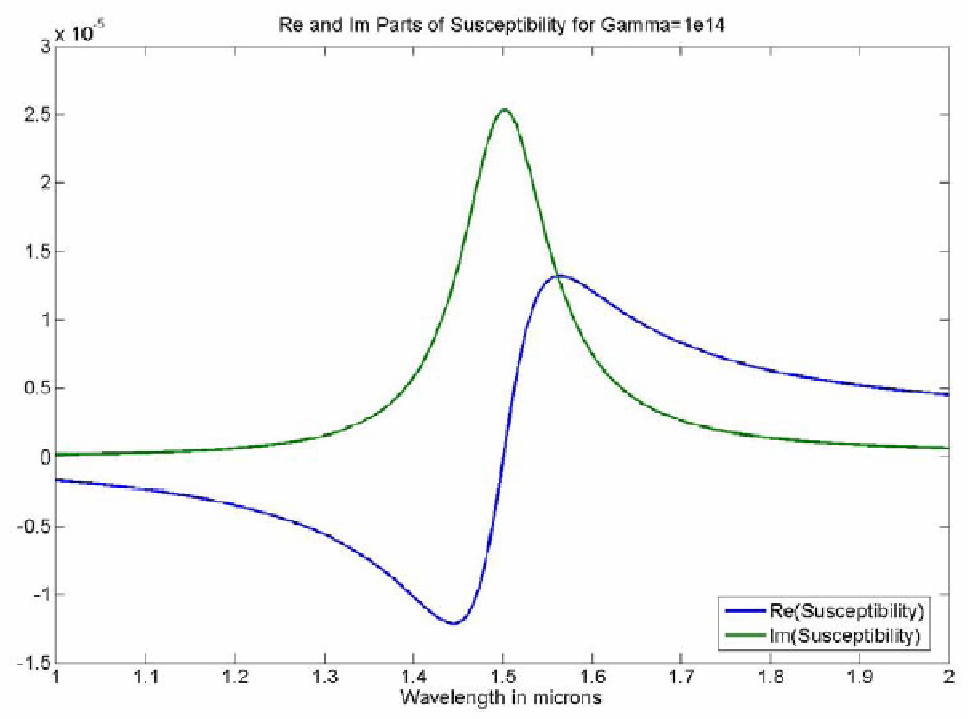
Absorption coefficient is given by: $\alpha(\lambda)=\frac{4\pi\kappa(\lambda)}{\lambda_0}$ (2.7)

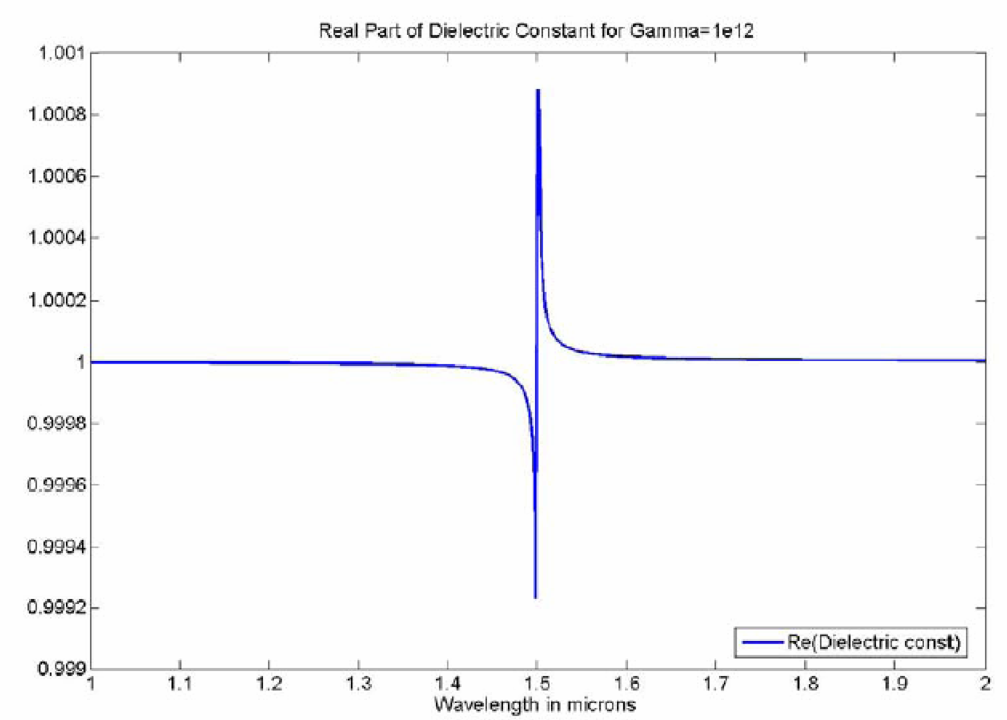
Punching in the numbers we get: $\omega_p=1.783\times10^{12}$, $\omega_0=1.256\times10^{15}$, we can plot the graphs now.

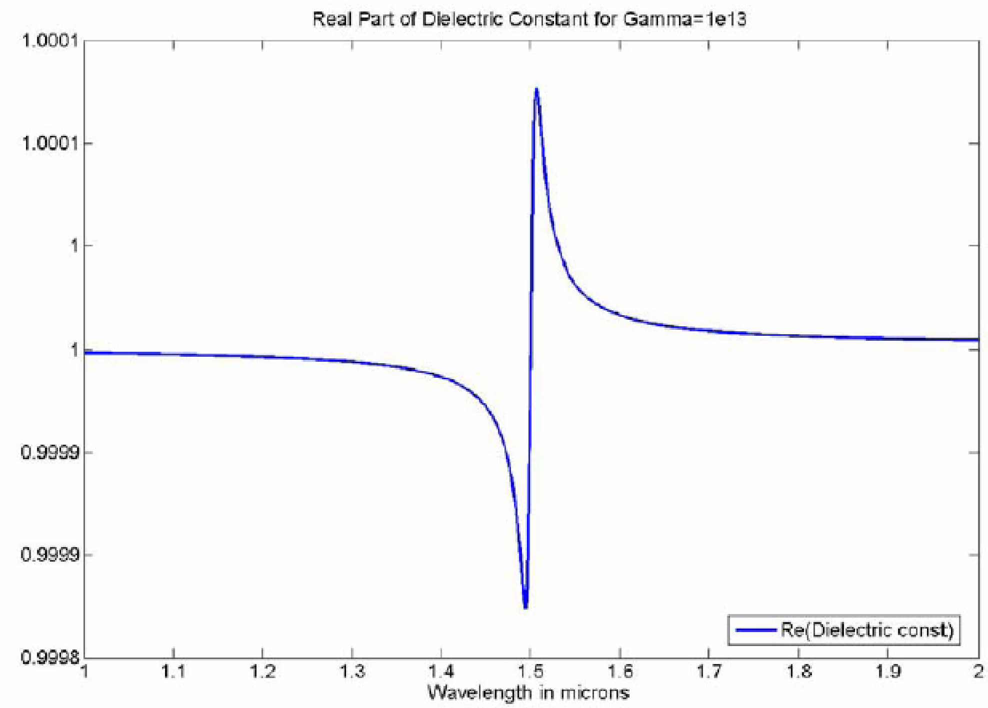
Please note that Imaginary parts of Susceptibility and Dielectric constants are same (equation 2.5) and hence aren’t plotted twice to avoid redundancy.

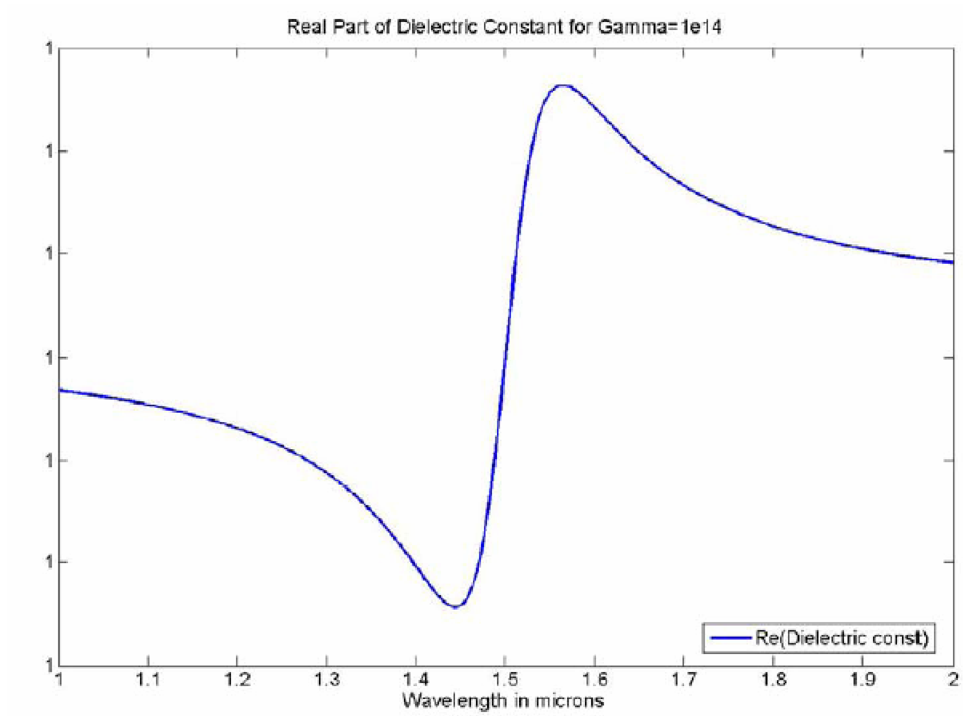


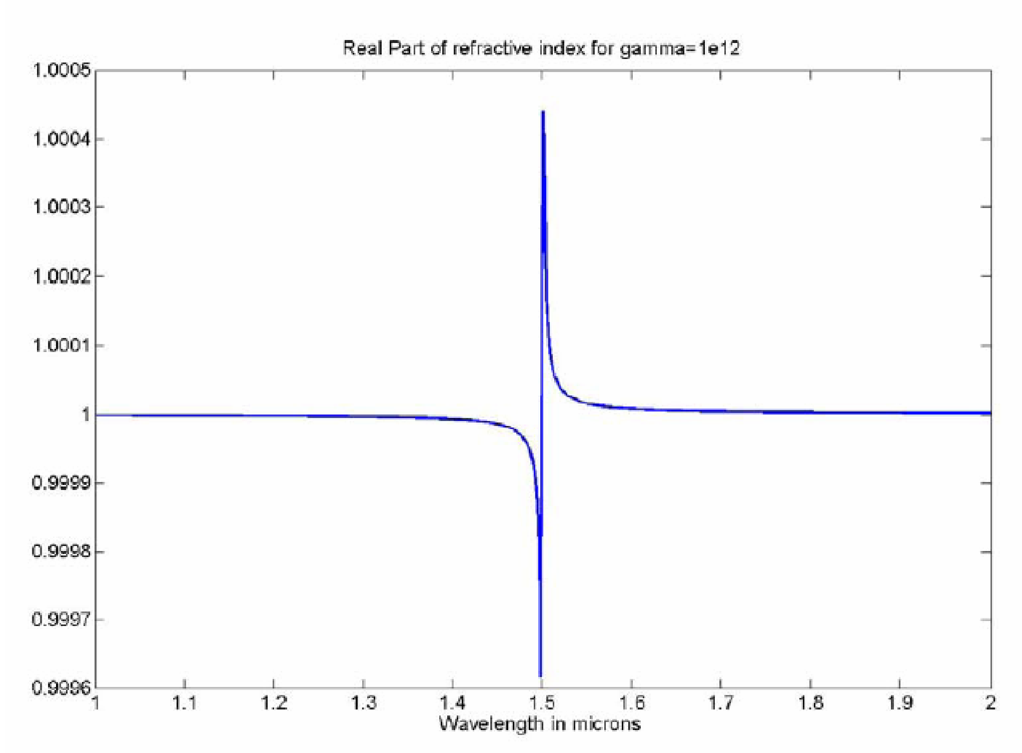


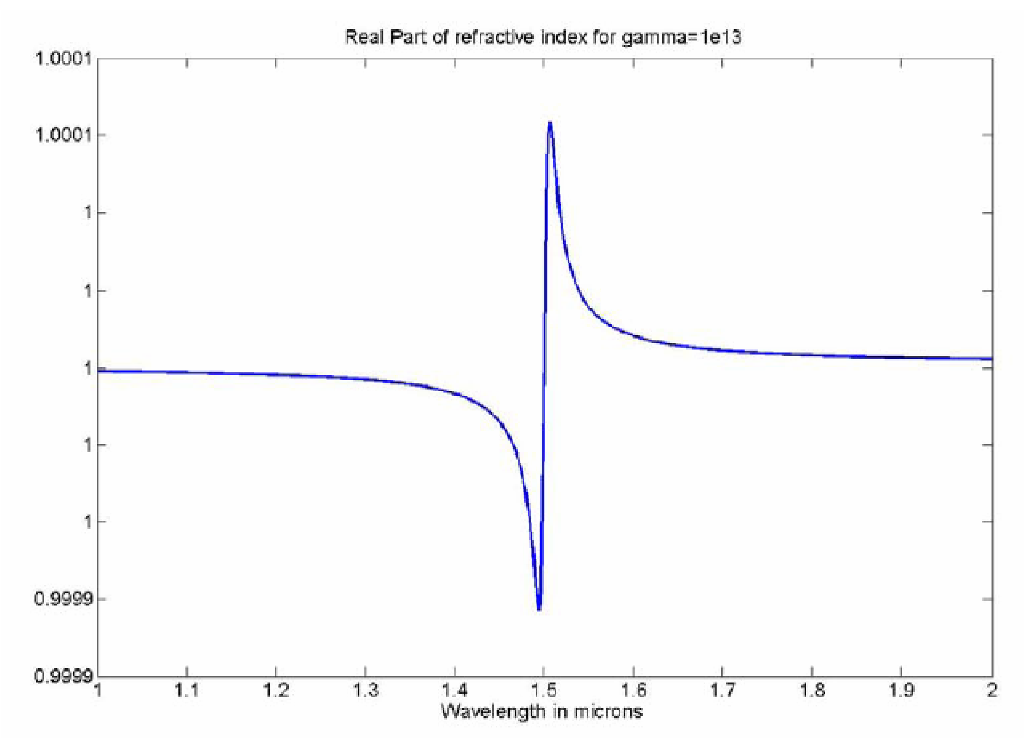

Re and Im Parts of Susceptibility for Gamma=1e14
x 10^-5
3
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
1
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2
Wavelength in microns
Re(Susceptibility)
Im(Susceptibility)

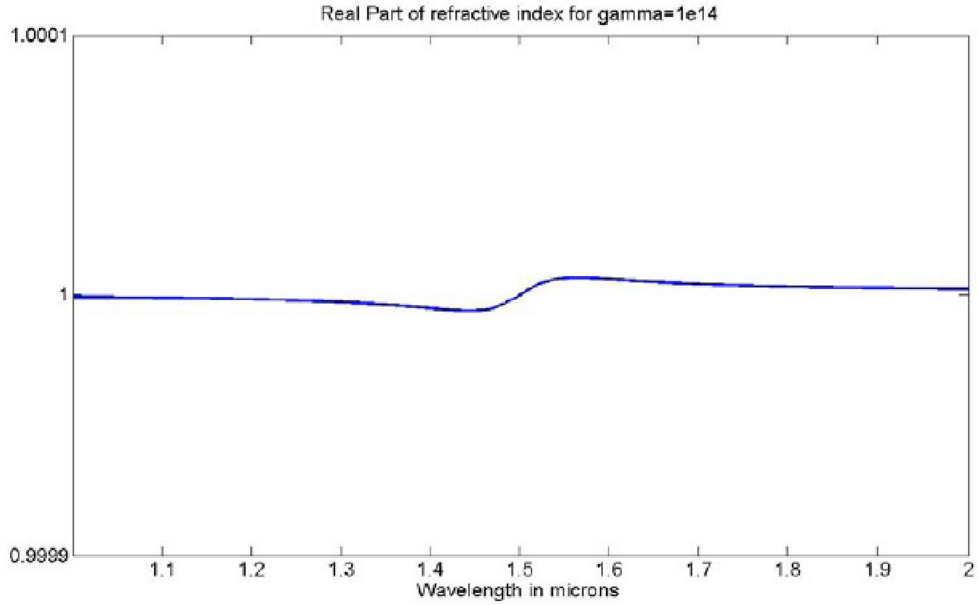

Real Part of Dielectric Constant for Gamma=1e12
1.001
1.0008
1.0006
1.0004
1.0002
1
0.9998
0.9996
0.9994
0.9992
0.999
1
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2
Wavelength in microns
Re(Dielectric const)


Real Part of Dielectric Constant for Gamma=1e13
1.0001
1.0001
1
1
0.9999
0.9999
0.9998
1
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2
Wavelength in microns
Re(Dielectric const)

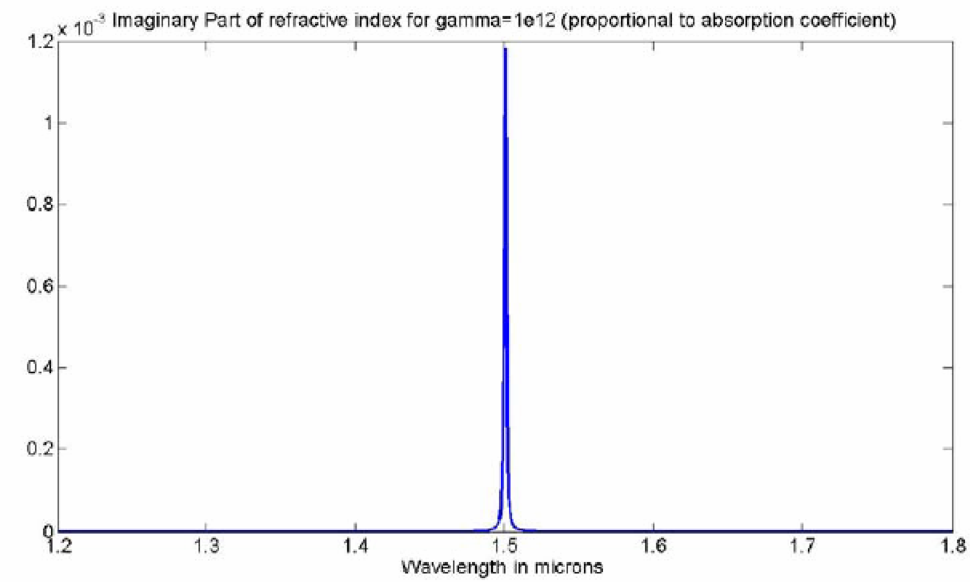

Real Part of Dielectric Constant for Gamma=1e14
1
1
1
1
1
1
1
1
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2
Wavelength in microns
Re(Dielectric const)


Real Part of refractive index for gamma=1e12
1.0005
1.0004
1.0003
1.0002
1.0001
1
0.9999
0.9998
0.9997
0.9996
1
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2
Wavelength in microns

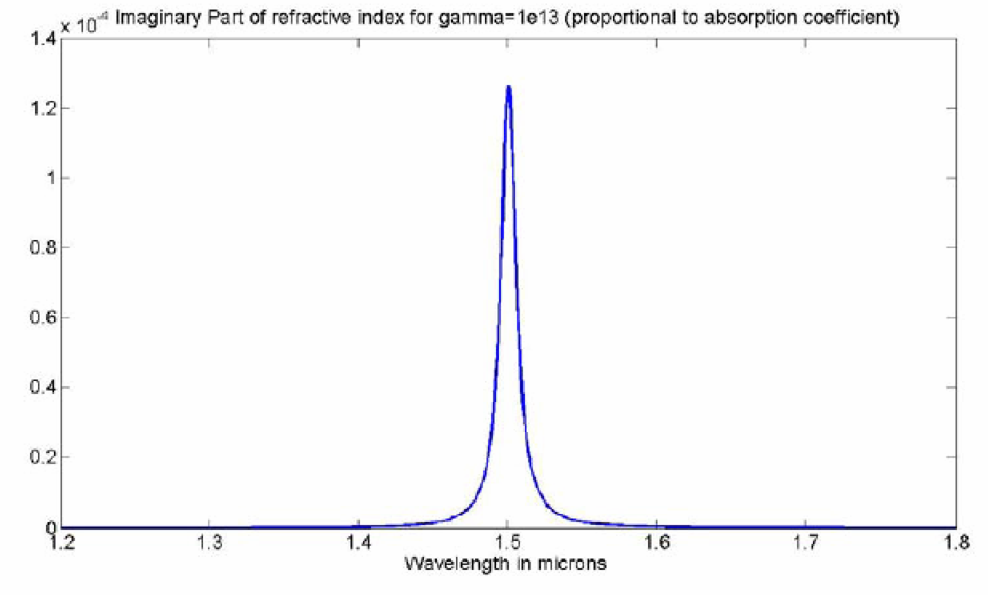

Real Part of refractive index for gamma=1e13
1.0001
1.0001
1
1
1
1
1
0.9999
0.9999
1
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2
Wavelength in microns

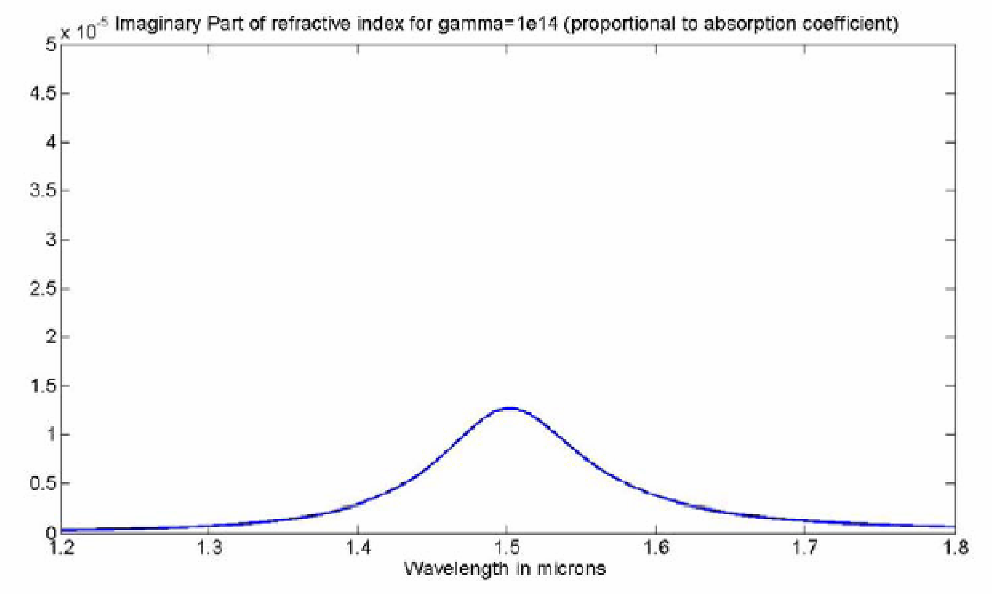


Real Part of refractive index for gamma=1e14
1.0001
1
0.9999
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2
Wavelength in microns



x 10^-3 Imaginary Part of refractive index for gamma=1e12 (proportional to absorption coefficient)
1.2
1
0.8
0.6
0.4
0.2
0
1.2
1.3
1.4
1.5
1.6
1.7
1.8
Wavelength in microns


x 10^-4 Imaginary Part of refractive index for gamma=1e13 (proportional to absorption coefficient)
1.4
1.2
1
0.8
0.6
0.4
0.2
0
1.2
1.3
1.4
1.5
1.6
1.7
1.8
Wavelength in microns


x 10^-5 Imaginary Part of refractive index for gamma=1e14 (proportional to absorption coefficient)
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
1.2
1.3
1.4
1.5
1.6
1.7
1.8
Wavelength in microns

Problem # 2. Plot the dipole radiative lifetime as a function of wavelength for 100nm < λ_0 <2000nm. (10 points)

$$\tau_R = \frac{6\pi\varepsilon_0 mc^3}{e^2\omega_0^2} \text{ (where, } n=1)$$

$$\omega_0 = 2\pi\frac{c}{\lambda_0} \quad (100\times10^{-9} \le \lambda_0 \le 2000\times10^{-9})$$

$$\varepsilon_0 = 8.85\times10^{-12}$$

$$m = 9.11\times10^{-31}$$

$$c = 3\times10^{8}$$

$$e = 1.6\times10^{-19}$$