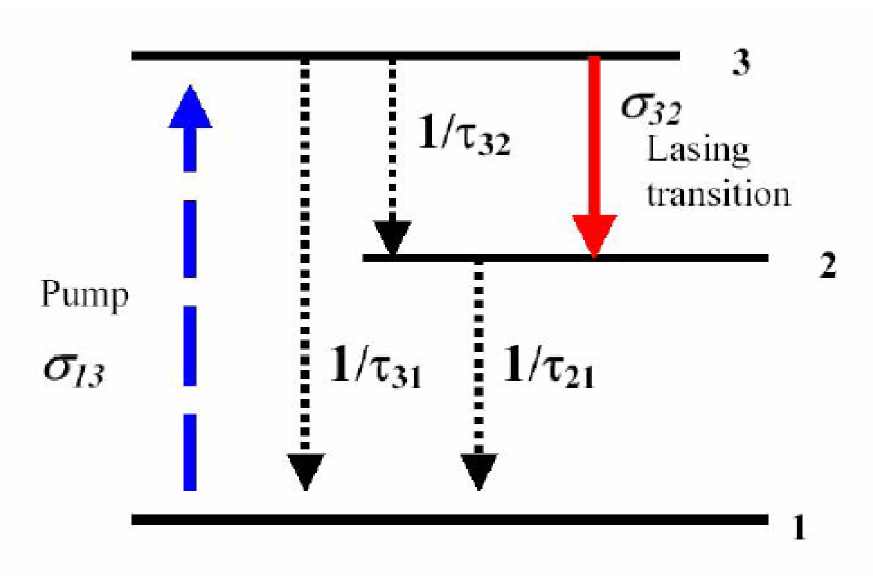
Introduction to Lasers (520.482)

Solutions: Homework 2



Points distribution: #1 and #3-7 (5 points each), #2 (20 points). Max = 50 points.

(1). $\dfrac{1}{\tau_3} = \dfrac{1}{\tau_{31}} + \dfrac{1}{\tau_{32}}; \quad \tau_2 = \tau_{21}$

(2). Steady state Rate Equations:

$$\frac{dN_3}{dt} = \frac{I_p\sigma_{13}}{h\nu_p}(N_1 - N_3) + N_3\left(\frac{1}{\tau_{31}} + \frac{1}{\tau_{32}}\right) = \frac{I_p\sigma_{13}}{h\nu_p}(N_1 - N_3) - N_3\left(\frac{1}{\tau_3}\right) = 0 \qquad (1)$$

$$\frac{dN_2}{dt} = \frac{N_3}{\tau_{32}} - \frac{N_2}{\tau_{21}} = 0\text{, implies} \Rightarrow \quad N_2 = N_3\frac{\tau_{21}}{\tau_{32}} \qquad (2)$$

But the total number of atoms is constant. So,

$$N = N_1 + N_2 + N_3\text{, hence } \therefore \quad N_1 = N - N_3\frac{\tau_{21}}{\tau_{32}} - N_3 \qquad (3)$$

Substituting (3) in (1) we get:

$$\frac{I_p\sigma_{13}}{h\nu_p}\left(N - N_3\frac{\tau_{21}}{\tau_{32}} - 2N_3\right) + N_3\left(\frac{1}{\tau_3}\right) = 0\text{; hence} \Rightarrow \quad N_3 = \frac{N\left(\dfrac{I_p\sigma_{13}}{h\nu_p}\right)}{\dfrac{I_p\sigma_{13}}{h\nu_p}\left(2 + \dfrac{\tau_{21}}{\tau_{32}}\right) + \left(\dfrac{1}{\tau_3}\right)} \qquad (4)$$

Find the population inversion density

$$\Delta N_{32} = N_3 - N_2 = N_3\left(1-\frac{\tau_{21}}{\tau_{32}}\right) = N\frac{\left(I_p\sigma_{13}/h\nu_p\right)\left(1-\frac{\tau_{21}}{\tau_{32}}\right)}{\left(I_p\sigma_{13}/h\nu_p\right)\left(2+\frac{\tau_{21}}{\tau_{32}}\right)+\frac{1}{\tau_3}}$$

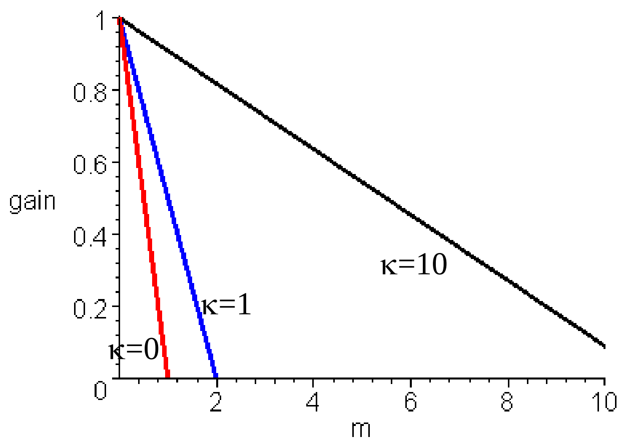
(3) The **sign** of population inversion depends only on the ratio of lifetimes τ_{21}/τ_{32}, and thus there is no transparency pump density (or you can say that it is equal to zero as long as $\tau_{21}/\tau_{32}<0$ and equal to infinity if $\tau_{21}/\tau_{32}>0$

(4) $\tau_2/\tau_3=m$ $\tau_{32}/\tau_{31}=\kappa$ $\tau_3^{-1} = \tau_{31}^{-1}+\tau_{32}^{-1} = (\kappa+1)\tau_{32}^{-1}$ $\tau_{21}/\tau_{32} = (\tau_2/\tau_3)(\kappa+1)^{-1} = m(\kappa+1)^{-1}$

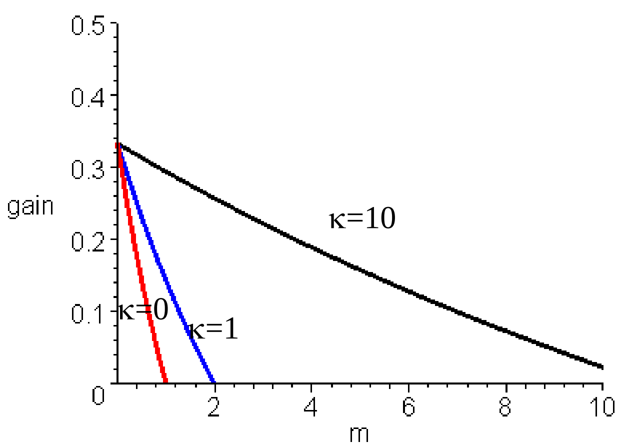
$$\Delta N_{32} = N\frac{\left(I_p\sigma_{13}/h\nu_p\right)\left(1-m(\kappa+1)^{-1}\right)}{\left(I_p\sigma_{13}\tau_3/h\nu_p\right)\left(2+m(\kappa+1)^{-1}\right)+1}$$

assume three different values of pump saturation, e.g.

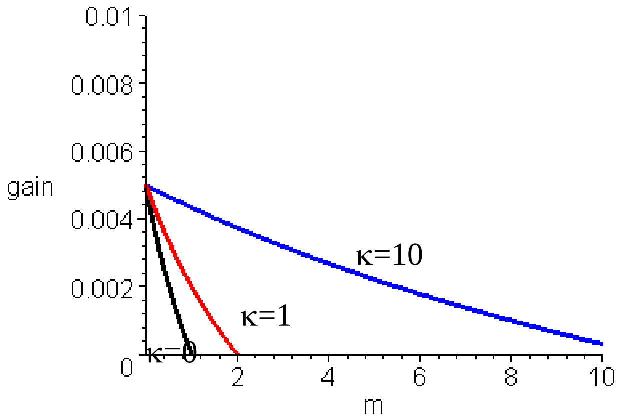
a. very low pump power $I_p\sigma_{13}\tau_3/h\nu_p << 1$ (gain is plotted in relative units)



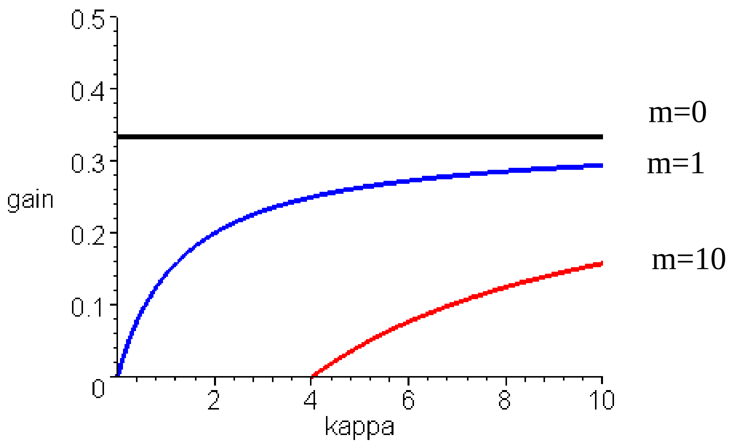
b. medium pump power $I_p\sigma_{13}\tau_3/h\nu_p = 1$ (gain is plotted in relative units)



c. very high pump power $I_p\sigma_{13}\tau_3/h\nu_p = 100$



(5) Now plot gain as a function of κ for fixed m for medium pump power $I_p \sigma_{13} \tau_3 / h\nu_p = 1$



(6) No, steady state gain can only be obtained if $\tau_{21}/\tau_{32}<0$.

(7) The answer is **yes.** If the pump pulse duration $\tau_p << \tau_3$ the pump pulse will be simply integrated and at the end of it the population of the upper laser level will be roughly

$$N_3(\tau_p) \approx I_p \tau_p \sigma_{21} / h\nu_p$$

Assume now the worst case scenario $\tau_{21} >> \tau_{32}$ – then we can solve the differential equation for the populations:

$$N_3(t) = N_3(\tau_p) e^{-(t-\tau_p)/\tau_3}$$

$$N_2(t) = \int_{\tau_p}^{t} \frac{1}{\tau_{32}} N_3(t) dt = \frac{\tau_3}{\tau_{32}} N_3(\tau_p)(1 - e^{-(t-\tau_p)/\tau_3})$$

$$\Delta N_{32}(t) = N_3(\tau_p)\left[(1+\frac{\tau_3}{\tau_{32}})e^{-(t-\tau_p)/\tau_3} - \frac{\tau_3}{\tau_{32}}\right] = \frac{I_p \sigma_{31} \tau_p}{h\nu_p}\left[\frac{\kappa+2}{\kappa+1} e^{-(t-\tau_p)/\tau_3} - \frac{1}{\kappa+1}\right]$$

Clearly gain exists for the time interval from τ_{pump} until $t_{end} \sim \tau_p + \tau_3 \ln \frac{1}{\kappa+2}$