Introduction to Lasers, ECE 520.482

A problem: laser spot on the Moon

The problem is formulated in terms of diffraction theory of Gaussian beams found in the class notes http://striky.ece.jhu.edu/˜sasha/COURSES/Gauss.diffr.pdf
You are given a Gaussian beam with the wavelength λ ***and of size (radius) a and plane phase front*** at the original point; assume propagation in vacuum.

Find out what is the radius $a = a_{opt}$ ***needed to attain a minimal size (radius)*** ρ_{min} ***of that beam at an arbitrary distance z from the original point along the axis of propagation; also, what is*** ρ_{min}***?***

Consider also a specific case of $\lambda = 1\ \mu m$***, and*** $z = 300,000\ km$***, i. e. a laser spot on the Moon.***

Solution:

Using eq. (4.29) from the above notes, we rewrite it as

$$\rho^2 = a^2 + z^2/k^2 a^2 , \quad \text{where} \quad k = 2\pi/\lambda \tag{1}$$

One can immediately see from (1) that for z and k fixed, ρ^2 reaches minimum at

$$a = a_{opt} \equiv \sqrt{z/k} = \sqrt{z\lambda/2\pi} \tag{2}$$

and thus

$$\rho_{min} = a_{opt}\sqrt{2} \equiv \sqrt{2z/k} = \sqrt{z\lambda/\pi} \tag{3}$$

In a perticular case of the Moon, we have $\rho_{min} \approx 10\ m$.